On-the-job Training and On-the-job Search: Wage-Training-Promotion in a Frictional Labor Market

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On-the-job Training and On-the-job Search

Introduction
Motivation

The Model
Primitives
Steady State Equilibrium

Numerical Analysis
Steady State
Comparative Statics

Conclusion

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Motivation

- Human Capital Accumulation (post schooling)
  - the major contributor of individual wage growth and economic growth
  - the first best outcome: workers should pay for the cost of general training.
  - In reality,
    - only firms can provide general training in many cases and workers cannot commit to staying with the training firms.

- Research Questions:
  - Does the ME provide the efficient level of training?
  - Does the ME provide more (or less) training, as search friction is mitigated?
Previous Literature

- Becker (1964): perfectly competitive labor market
  - Under perfect competition, firms provide the efficient level of training, and workers pay the training cost through low wages during training.

- Acemoglu (1997): frictional labor market
  - incorporates job-to-job transition into an ex-post bargaining model
  - positive externality for subsequent poaching firms (free rider problem)

- Sanders and Taber (2012)
  - under-investment on general human capital
  - over-investment on job specific human capital

- This paper
  - As the offer arrival rate increases, ...
  - first mover advantage? (general training but specific asset?)
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The Basic Model

- Burdett and Mortensen (1998) with productivity differentials
- a unit measure of risk neutral (lifetime income maximizing) workers
  - A newly born worker enters the labor market as unskilled and unemployed.
  - The unemployed worker gets employed at rate $\lambda^0$.
  - The employed worker finds another job at rate $\lambda^1$ and gets laid off at rate $\delta$.
  - The unskilled worker acquires (general) skills at rate $\mu x$ through training.
  - All workers retire at rate $\rho$ and they are replaced with newly born workers.
- a unit measure of heterogenous firms ($p \sim H(p)$)
  - Each firm maintains one vacancy at every instant.
  - The recruiting firm with $p$ posts ($w_s(p), (w_u(p), x(p), w_t^s(p))$).

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The Basic Model

- Unemployed Workers
  - retire at rate $\rho$, and get employed at rate $\lambda^0$.

  $$r_{U_i} = b - \rho U_i + \lambda^0 \int \max\{z - U_i, 0\} dF_i(z), \text{ for each } i \in \{u, s\}$$

- Skilled Employed Workers
  - retire at rate $\rho$, get laid off at rate $\delta$, and find offers at rate $\lambda^1$.

  $$r_{E_s}(p) = w_s(p) - \rho E_s(p) + \delta(U_s - E_s(p)) + \lambda^1 \int \max\{z - E_s(p), 0\} dF_s(z)$$

- Unskilled Employed Workers
  - retire at rate $\rho$, get laid off at rate $\delta$, find offers at rate $\lambda^1$, and
  - acquire (general) skills at rate $\mu x(p)$.

  $$r_{E_u}(p) = w_u(p) - \rho E_u(p) + \delta(U_u - E_u(p)) + \lambda^1 \int \max\{z - E_u(p), 0\} dF_u(z)$$

  $$+ \mu x(p)(E^t_s(p) - E_u(p))$$
The Basic Model

- Operating Firms with Skilled Matches
  - deliver the committed values through ...

\[ rJ_s(E_s(p), p) = p + s - w_s(p) - [\rho + \delta + \lambda^1(1 - F_s(E_s(p)))]J_s(E_s(p), p) \]

- Operating Firms with Unskilled Matches
  - deliver the committed values through ...

\[ rJ_u(E_u(p), p) = \max_{w_u, x, E_s^t} p - w_u - c(x) - [\rho + \delta + \lambda^1(1 - F_u(E_u(p)))]J_u(E_u(p), p) \]

subject to the promise-keeping constraint on \( E_u(p) \).

- F.O.C.
  - \( w_s^t(p) = p + s \)
  - \( c'(x) = \mu(E_s^t(p) - E_u(p) - J_u(p)) \)
  - The promise keeping constraint determines unskilled wages.
The Basic Model

- Recruiting Firms
  - post \((E_u(p), E_s(p))\) to maximize

\[
\begin{align*}
[\lambda^0 u_s + \lambda^1 G_s(E_s)]J_s(E_s, p) + [\lambda^0 u_u + \lambda^1 G_u(E_u)]J_u(E_u, p)
\end{align*}
\]
The Basic Model

Given firms’ productivity distribution $H(p)$, a steady state equilibrium with on-the-job training and on-the-job search consists of value equations $\{U_i, E_i, J_i\}$ compensation packages $\{(w_u(p), x(p), E^t_s(p)), (w_s(p))\}$ and steady state measures $\{F_i, G_i, u_i\}$ that jointly satisfy the following conditions.

(i) Given $F_i$, workers make optimal job turnover decision.

(ii) Given $\{F_i, E_i\}$, operating firms optimally deliver the committed values.

(iii) Given $\{G_i, u_i\}$, recruiting firms post their contract to maximize their profit.

(iv) $\{F_i, G_i, u_i\}$ are stationary and consistent with the behavior of each agents.
Steady State Equilibrium

Figure: Equilibrium Support of Wages
Steady State Equilibrium

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Steady State Equilibrium

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Steady State Equilibrium

Figure: Equilibrium Support of Wages
Steady State Equilibrium

**Figure:** Equilibrium Support of Wages
Steady State Equilibrium

Figure: Equilibrium Support of Wages
STEADY STATE EQUILIBRIUM

Figure: Equilibrium Support of Wages
Steady State Equilibrium

- Proposition 1  The optimal training intensity is characterized by
  \[ c'(x(p))(r + \rho + \delta)/\mu + x(p)c'(x(p)) - c(x(p)) = s + \delta(U_s - U_u) \]
  \[ + \lambda^1 \int_{E_s^t(p)}[z - E_s^t(p)]dF_s(z) - \lambda^1 \int_{E_u(p)}[z - E_u(p) - J_u(p)]dF_u(z) \]

- Training intensity \( x(p) \) is well-defined.
- The right hand side is (more likely to be) positive.
**Steady State Equilibrium**

- Constrained Social Planner
  - maximizes the present value of the expected output flow throughout the life of a newly born worker in the steady state equilibrium.

\[
(r + \rho)S_s^*(p) = p + s + \delta(U_s^* - S_s^*) + \lambda^1 \int_P^P [S_s^*(p') - S_s^*(p)] dH(p')
\]

\[
(r + \rho)S_u^*(p) = p - c(x^*(p)) + \delta(U_u^* - S_u^*) + \mu x^*(p)(S_s^*(p) - S_u^*(p))
\]

\[
+ \lambda^1 \int_P^P [S_u^*(p') - S_u^*(p)] dH(p')
\]

\[
(r + \rho)U_i^*(p) = b + \lambda^0 \int_P^P [S_i^*(p') - U_i^*(p)] dH(p')
\]

- chooses the training intensity such that \( c'(x^*(p)) = \mu(S_s^*(p) - S_u^*(p)) \)

\( (vs \ c'(x(p)) = \mu(E_s^t(p) - E_u(p) - J_u(p))) \)
Steady State Equilibrium

- Proposition 2  The efficiency benchmark implies that for each \( p \in [p, \bar{p}] \)

\[
c'(x^*(p))(r + \rho + \delta)/\mu + x^*(p)c'(x^*(p)) - c(x^*(p)) = s + \delta(U_s^* - U_u^*)
\]

(i) \( dx^*(p)/dp = 0 \)

(ii) \( dx^*(p)/d\lambda^1 = 0 \) and \( dx^*(p)/d\lambda^0 > 0 \)
Steady State Equilibrium

- In the market equilibrium,

\[
c'(x(p))(r + \rho + \delta)/\mu + x(p)c'(x(p)) - c(x(p)) = s + \delta(U_s - U_u) \\
+ \lambda^1 \int_{E_s^t(p)} [z - E_s^t(p)]dF_s(z) - \lambda^1 \int_{E_u(p)} [z - E_u(p) - J_u(p)]dF_u(z)
\]

- In the social planner’s problem,

\[
c'(x^*(p))(r + \rho + \delta)/\mu + x^*(p)c'(x^*(p)) - c(x^*(p)) = s + \delta(U_s^* - U_u^*)
\]
Steady State Equilibrium

- Consider an alternative environment for a while.
  - Workers retire at rate $\delta$ and all retirees due to $\delta$-shock are replaced with newly born unemployed workers possessing the same skills.
  - Simply, subtract $\delta U_i$ from all value equations.
  - Designate by overhead $\tilde{}$ all endogenous variables associated with the alternative environment.
**Steady State Equilibrium**

- In the market equilibrium,

\[ c'(\tilde{x}(p))(r + \rho + \delta)/\mu + \tilde{x}(p)c'(\tilde{x}(p)) - c(\tilde{x}(p)) = s \]

\[ + \lambda^1 \int_{\tilde{E}_s(p)}^{\tilde{E}_t(p)} [z - \tilde{E}_s(t)] d\tilde{F}_s(z) - \lambda^1 \int_{\tilde{E}_u(p)}^{\tilde{E}_t(p)} [z - \tilde{E}_u(t) - \tilde{J}_u(p)] d\tilde{F}_u(z) \]

- In the social planner’s problem,

\[ c'(\tilde{x}^*(p))(r + \rho + \delta)/\mu + \tilde{x}^*(p)c'(\tilde{x}^*(p)) - c(\tilde{x}^*(p)) = s \]

- No distortion at the top (without separation shocks)
- Allocation inefficiency: deterring socially efficient job-to-job transition
- Training inefficiency: over-investment by firms with higher productivity
**Steady State Equilibrium**

Figure: Equilibrium Support of Wages
**Steady State Equilibrium**

**Figure:** Equilibrium Support of Wages
Steady State Equilibrium

(a) Without Separation Shocks ($\delta = 0$)  
(b) With Separation Shocks ($\delta > 0$)

Figure: Training Intensity
**On-the-job Training and On-the-job Search**

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**Conclusion**
## Baseline Simulation

### Table 1: Parameter Values

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<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>([p + s, \bar{p} + s] = [1.0, 2.0])</td>
<td>the productivity support</td>
<td></td>
</tr>
<tr>
<td>(s = 0.25)</td>
<td>productivity improvement through training</td>
<td></td>
</tr>
<tr>
<td>(\eta = 1.0)</td>
<td>the shape parameter of (H(p))</td>
<td></td>
</tr>
<tr>
<td>(\gamma = 2.0)</td>
<td>cost function parameter</td>
<td></td>
</tr>
<tr>
<td>(r = 0.012)</td>
<td>interest rate</td>
<td></td>
</tr>
<tr>
<td>(\rho = 0.008)</td>
<td>retirement rate</td>
<td></td>
</tr>
<tr>
<td>(\delta = 0.064)</td>
<td>separation rate</td>
<td></td>
</tr>
<tr>
<td>(\lambda^0 = 1.35)</td>
<td>job finding rate by unemployed workers</td>
<td></td>
</tr>
<tr>
<td>(\lambda^1 = 0.45)</td>
<td>job finding rate by employed workers</td>
<td></td>
</tr>
<tr>
<td>(b = 0.16)</td>
<td>unemployment benefit</td>
<td></td>
</tr>
</tbody>
</table>

Productivity Distribution:  \[ H(p) = \frac{1 - (p/\bar{p})^\eta}{1 - (\rho/\bar{p})^\eta} \]

Cost function:  \[ c(x) = x^\gamma \]
Baseline Simulation

(a) Workers’ Values

(b) Firms’ Values

(c) Wages

(d) The Value-earning Distribution

Figure 3: The Baseline Simulation Result
**Baseline Simulation**

Figure 4: Training Intensity

<table>
<thead>
<tr>
<th></th>
<th>unskilled workers</th>
<th>skilled workers</th>
<th>training cost</th>
<th>total output</th>
<th>Net output</th>
</tr>
</thead>
<tbody>
<tr>
<td>ME</td>
<td>0.6001</td>
<td>0.3492</td>
<td>0.0159</td>
<td>1.3859</td>
<td>1.3700</td>
</tr>
<tr>
<td>PP</td>
<td>0.6027</td>
<td>0.3467</td>
<td>0.0156</td>
<td>1.3862</td>
<td>1.3707</td>
</tr>
<tr>
<td>ME/PP</td>
<td>0.9958</td>
<td>1.0073</td>
<td>1.0191</td>
<td>0.9997</td>
<td>0.9995</td>
</tr>
</tbody>
</table>

**Table 2: The Outcome of the Baseline Simulation**

ME: the market equilibrium outcome  
PP: the planner’s solution  
ME/PP: the ratio of ME to PP
COMPARATIVE STATICS

Figure 6: Training Intensity
## Introduction

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## Numerical Analysis

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## Conclusion
CONCLUSION

- This paper studies the coexistence of on-the-job training and search.
  - Key Innovation
    - including on-the-job search by unskilled workers
    - allowing (initial) skill-dependent back-loading contracts
  
  - Key Channel:
    - The back-loaded compensation works, as if it were relationship-specific asset.

  - Key Implication:
    - The market equilibrium outcome, relative to the planner’s outcome, get more skilled workers (training inefficiency) but less output (allocation inefficiency).
    - The aggregate efficiency loss is moderate (offsetting the positive externality).
    - The efficiency loss can be resolved, as search friction is mitigated.
Thanks for listening!

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