

# On the Number and Size of Banks: Efficiency and Equilibrium

Angela Kui Huang

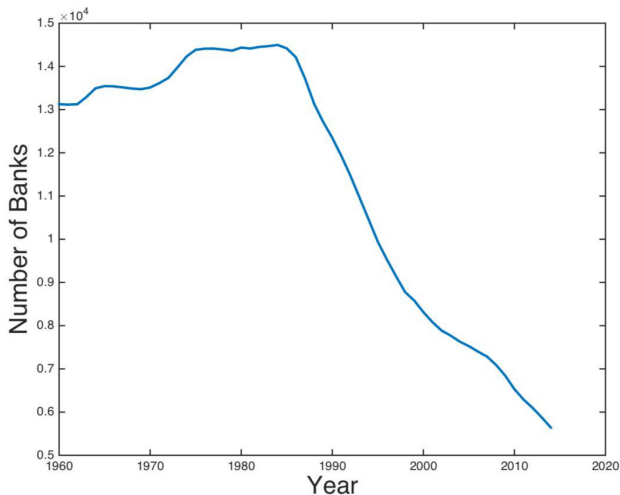
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# Outline of the Talk

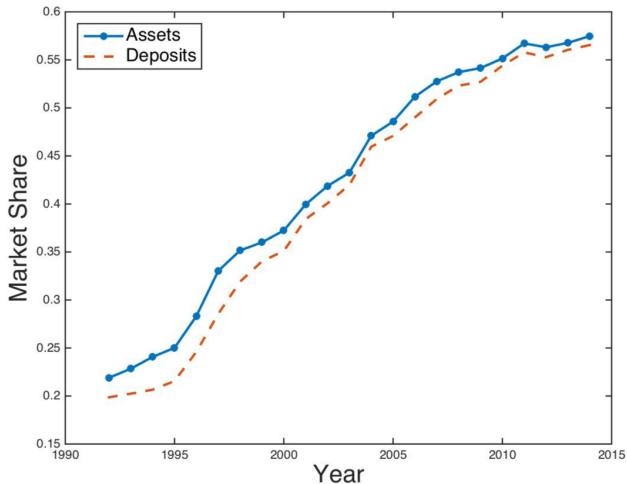
- 1 Motivation
- 2 Related Literature
- 3 Model
  - Environment without Banking
  - Environment with Banking
- 4 Efficiency
- 5 Equilibrium
- 6 Conclusions

Figure 1a: Drop in the number of banks in the United States



# Motivation: Continued

Figure 1b: Market share of 10 largest banks in the United States



# Motivation: Continued

- Questions:
  - Under what conditions is it socially optimal to have few large banks versus many small banks?
  - Why don't we want too few or too many banks?
  - Is “unfettered competition” in banking optimal?
- Goal: develop a theoretical model where banking arises endogenously, to address these questions.

- Ingredients in the theory:
  - Frictions: limited commitment and imperfect monitoring
  - Banking arises endogenously because of economies of scale.
  - The planner designates a fraction of ex-ante homogenous agents to be bankers, and concentrates monitoring efforts on them.

- Cost-Benefit Trade-off:
  - having fewer bankers reduces total monitoring costs,
  - but this means more deposits per banker,
  - having more deposits, however, increases the bankers' incentives to divert deposits for their own profit.

- Results:
  - The planner needs to give the bankers some reward to dissuade such opportunistic behavior.
  - The optimal number of banks is negatively related to the fixed and marginal monitoring costs, impatience, the temptation to default, but positively related to the rate of return.
  - When the tax on banks is not too high, there exists a non-degenerate stable stationary equilibrium.
  - The equilibrium allocation is optimal only if the government limits entry of banks, by charging a tax on the bankers or rationing bank charters.



- Credit with Limited Commitment
  - Kehoe and Levine (1993), Alvarez and Jermann (2000), Gu et al. (2013a)
- Banking
  - Leland and Pyle (1977), Boyd and Prescott (1986), Diamond and Dybvig (1983)
  - Gu et al. (2013b), Cavalcanti and Wallace (1999a, 1999b), He et al. (2005, 2008)
  - Janicki and Prescott (2006), Corbae and D'Erasmus (2013)
- Monitoring
  - Diamond (1984), Williamson (1986,1987), Broecker (1990), Holmstrom and Tirole (1997), Diamond and Rajan( 2001)

# Model

## Environment without Banking

- Time is discrete and continues forever. Each period is divided into two subperiods.
- Two types of agents: type 1 and type 2 .
- Type 1 agents consume good  $x$  and produce good  $y$ ; type 2 agents consume good  $y$  and produce good  $x$ .
- Both goods are produced in the first subperiod; good  $x$  is consumed in the first subperiod, while good  $y$  is consumed in the second.

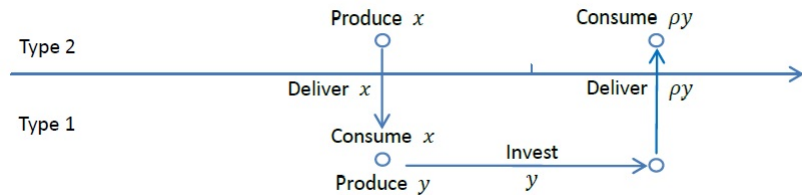
# Model

## Environment without Banking: Continued

- Type 1 agents invest good  $y$  across subperiods, with a fixed return  $\rho$ .
- Type 2 agents cannot invest for themselves.
- Utility of type 1 is  $U^1(x, y)$ , and utility of type 2 is  $U^2(\rho y, x)$ .

# Model

## Environment without Banking: Timeline



- Frictions:

- ① Limited Commitment

- When type 1 agents are supposed to deliver the goods, they can renege to obtain a payoff  $\lambda\rho y$  over and above  $U^1(x, y)$ .

- ② Imperfect monitoring

- Any deviation is detected by the mechanism with probability  $\pi$ , and punished with future autarky
    - $\pi$  is endogenous.

# Model

## Environment without Banking: Ways to Rationalize Monitoring

- Imperfect record keeping
- Deviation is observable but is costly to verify
- Type 1 defaults
  - ⇒ type 2 knows it
  - ⇒ type 2 communicates with the mechanism
  - ⇒ with probability  $\pi$ , the mechanism records it
  - ⇒ the deviator is punished to future autarky

# Model

## Environment without Banking: Monitoring Cost

- Type 2 needs to pay the monitoring cost

$$c(\pi) = \begin{cases} k_0 + \pi k & \text{if } \pi > 0 \\ 0 & \text{if } \pi = 0 \end{cases}$$

This implies increasing returns to scale.

# Model

## Environment without Banking: Repayment Constraint

- Repayment constraint for type 1 agent

$$U^1(x, y) + \beta V^1(x, y) \geq U^1(x, y) + \lambda \rho y + (1 - \pi) \beta V^1(x, y)$$

where  $V^1(x, y) = U^1(x, y) / (1 - \beta)$

- Notice  $U^1(x, y)$  is sunk at the time of decision.
- All of the analysis here focuses on stationary allocations.



# Model

## Environment with Banking

- The planner designates measure  $\mu$  of type 1 to be bankers, and concentrates monitoring efforts on them.
- The other  $1 - \mu$  of type 1 are non-bankers.
- The bankers and non-bankers are ex-ante homogeneous.

# Model

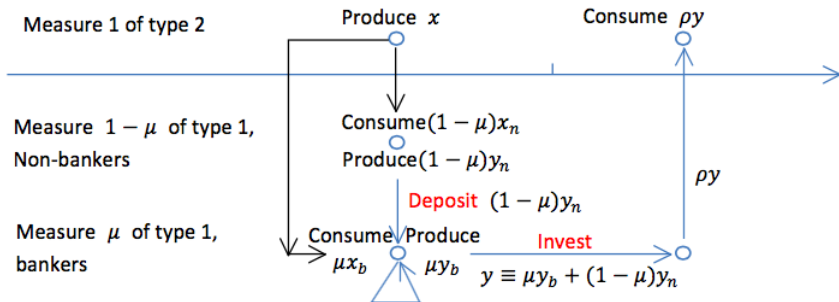
## Environment with Banking

- Type 1 non-bankers deposit their production with type 1 bankers.
- Bankers invest the combined good  $y$ , from their own production and the deposits, across subperiods.
- The size (assets) of each bank after investment is  $\rho y / \mu$ .

- Cost-benefit trade-off:
  - having fewer bankers reduces total monitoring costs,
  - but this means more deposits per banker,
  - having more deposits increases the bankers' incentives to divert deposits.

# Model

## Environment with Banking: Timeline



# Model

## Environment: Role of Banks

- A banker in the model has three features:
  - he takes deposits,
  - he makes investments on behalf of depositors,
  - his liabilities (claims on deposits) facilitate third-party transactions.
- We downplay other functions of banks, such as providing liquidity insurance or information processing.

# Model

## Environment with Banking: Continued

- Utility of banker is  $U^1(x_b, y_b)$ , utility of non-banker is  $U^1(x_n, y_n)$ , and utility of type 2 is  $U^2(\rho y, x) - \mu c(\pi)$ .
- Feasibility Constraints:

$$\mu x_b + (1 - \mu) x_n = x$$

$$\mu y_b + (1 - \mu) y_n = y$$

- A planner or mechanism recommends an incentive feasible allocation.
- Welfare

$$W(x_b, y_b, x_n, y_n, x, y, \mu, \pi) = \theta \left[ \mu U^1(x_b, y_b) + (1 - \mu) U^1(x_n, y_n) \right] \\ + (1 - \theta) \left[ U^2(\rho y, x) - \mu (k_0 + \pi k) \right]$$

where we put the same weight  $\theta$  on type 1 banker and non-banker, and weight  $1 - \theta$  on type 2 agent.

- Participation constraints:

$$U^1(x_b, y_b) \geq 0,$$

$$U^1(x_n, y_n) \geq 0,$$

$$U^2(\rho y, x) - \mu(k_0 + \pi k) \geq 0.$$

- Repayment constraint for the banker

$$U^1(x_b, y_b) + \beta V^1(x_b, y_b) \geq U^1(x_b, y_b) + \lambda \rho y / \mu + (1 - \pi) \beta V^1(x_b, y_b)$$



- The repayment constraint reduces to

$$U^1(x_b, y_b) \geq r\lambda\rho y / \pi\mu,$$

where  $r = (1 - \beta) / \beta$ .

- A high  $r$  or high  $\lambda$  both increase the temptation to default.
- An agent is more trustworthy when he has smaller  $r\lambda$ .

## Planner's Problem

$$\begin{aligned} & \max_{(x_b, y_b, x_n, y_n, x, y, \pi, \mu)} W \\ \text{s.t.} \quad & \mu x_b + (1 - \mu) x_n = x \\ & \mu y_b + (1 - \mu) y_n = y \\ & U^1(x_n, y_n) \geq 0 \\ & U^2(\rho y, x) - \mu(k_0 + \pi k) \geq 0 \\ & U^1(x_b, y_b) \geq r\lambda\rho y / \pi\mu \end{aligned}$$

## Lemma 1

The repayment constraint must bind,  $U^1(x_b, y_b) = r\lambda\rho y/\pi\mu$ .

- Suppose

$$U^1(x_b, y_b) = u(x_b) - y_b,$$

$$U^1(x_n, y_n) = u(x_n) - y_n,$$

$$U^2(\rho y, x) = \rho y - v(x).$$

- Lemma 1 implies  $\pi = \frac{r\lambda\rho y}{[u(x_b) - y_b]\mu}$ .

# Efficiency: Continued

Using  $\pi = \frac{r\lambda\rho y}{[u(x_b) - y_b]\mu}$ ,  $x = \mu x_b + (1 - \mu)x_n$ , and  $y_n = \frac{y - \mu y_b}{1 - \mu}$ , planner's problem becomes

$$\max_{(x_b, y_b, x_n, y, \mu)} W$$

$$\text{FOCs} \quad u'(x_b) - \frac{1-\theta}{\theta} v'(x) + \frac{1-\theta}{\theta} \frac{r\lambda\rho y k u'(x_b)}{\mu [u(x_b) - y_b]^2} = 0$$

$$y_b^* = 0$$

$$u'(x_n) - \frac{1-\theta}{\theta} v'(x) = 0$$

$$-1 + \frac{1-\theta}{\theta} \left[ \rho - \frac{r\lambda\rho k}{[u(x_b) - y_b]} \right] = 0 \Rightarrow x_b^* = \bar{x}_b(k, r, \lambda, \rho)$$

$$u(x_b) - u(x_n) - \frac{1-\theta}{\theta} v'(x) (x_b - x_n) - \frac{1-\theta}{\theta} k_0 = 0$$

## Proposition 1

$$x_b^* > x_n^*, \text{ and } y_b^* = 0.$$

The planner needs to give the bankers some reward to dissuade opportunistic behavior.

## Lemma 2

$$\frac{\partial x_b^*}{\partial k} > 0, \frac{\partial x_b^*}{\partial r} > 0, \frac{\partial x_b^*}{\partial \lambda} > 0, \text{ and } \frac{\partial x_b^*}{\partial \rho} < 0.$$

## Lemma 3

Suppose  $U^1(x, y) = \frac{x^{1-\alpha}-1}{1-\alpha} - y$ , where  $\alpha > 0$ , we have  $\frac{\partial(x_b^* - x_n^*)}{\partial k} > 0$ ,  
 $\frac{\partial(x_b^* - x_n^*)}{\partial r} > 0$ ,  $\frac{\partial(x_b^* - x_n^*)}{\partial \lambda} > 0$ ,  $\frac{\partial(x_b^* - x_n^*)}{\partial \rho} < 0$ .

## Proposition 2

$\frac{\partial \mu^*}{\partial k_0} < 0$ ,  $\frac{\partial \mu^*}{\partial k} < 0$ ,  $\frac{\partial \mu^*}{\partial r} < 0$ ,  $\frac{\partial \mu^*}{\partial \lambda} < 0$ ,  $\frac{\partial \mu^*}{\partial \rho} > 0$ .

# Equilibrium

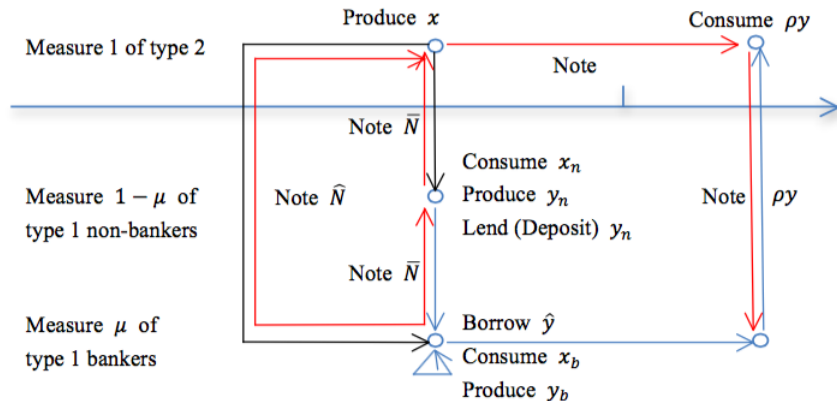
## Walrasian Pricing Mechanism

- To implement efficient allocations in decentralized competitive markets, there is a tension between equilibrium with free entry and having positive bank profit for incentive reasons.
- To improve efficiency, the government needs to limit entry of banks.
- One way is to charge a tax  $\tau$  on the bankers and give a transfer  $t$  to the non-bankers.
- Another way is to impose a quota by limiting the number of bank charters.



# Equilibrium

## Charging a Tax: Timeline



# Equilibrium

## Maximization Problem for Banker

- Repayment constraint for banker

$$\lambda\rho(\hat{y}_t + y_{bt}) + (1 - \Pi)\beta V_{bt+1}^1 \leq \beta V_{bt+1}^1$$

- Bellman equation for banker

$$V_{bt}^1 = U^1(x_{bt}, y_{bt}) + \beta V_{bt+1}^1$$

- Defining  $\phi_t \equiv \frac{\beta\Pi}{\lambda\rho} V_{bt+1}^1$  as the debt limit, repayment constraint becomes

$$\hat{y}_t + y_{bt} \leq \phi_t$$

- Bellman equation becomes

$$\phi_{t-1} = \frac{\beta\Pi}{\lambda\rho} U^1(x_{bt}, y_{bt}) + \beta\phi_t$$

- Maximization problem for banker

$$\begin{aligned} & \max_{(x_{bt}, y_{bt}, \hat{y}_t)} u(x_{bt}) - y_{bt} - \tau \\ \text{s.t.} \quad & p_{xt}x_{bt} + p_{yt}\hat{y}_t = \rho(\hat{y}_t + y_{bt}) \\ & \hat{y}_t + y_{bt} \leq \phi_t \end{aligned}$$

# Equilibrium

## Maximization Problem for Type 1 Non-banker and Type 2

- Maximization problem for type 1 non-banker

$$\begin{aligned} & \max_{(x_{nt}, y_{nt})} u(x_{nt}) - y_{nt} + t \\ \text{s.t.} \quad & p_{xt}x_{nt} = p_{yt}y_{nt} \end{aligned}$$

- Maximization problem for type 2

$$\begin{aligned} & \max_{(x_t, y_t)} \rho y_t - v(x_t) - \mu_t(k_0 + \Pi k) \\ \text{s.t.} \quad & \rho y_t = p_{xt}x_t \end{aligned}$$

# Equilibrium

## Free Entry

Free entry

$$\begin{aligned}\mu_t = 0 & \text{ if } u(x_{bt}) - y_{bt} - \tau < u(x_{nt}) - y_{nt} + t \\ \mu_t \in (0, 1) & \text{ if } u(x_{bt}) - y_{bt} - \tau = u(x_{nt}) - y_{nt} + t \\ \mu_t = 1 & \text{ if } u(x_{bt}) - y_{bt} - \tau > u(x_{nt}) - y_{nt} + t\end{aligned}$$

# Equilibrium

## Equilibrium

### Definition

An equilibrium is  $\{x_{bt}^e, y_{bt}^e, \hat{y}_t^e, x_{nt}^e, y_{nt}^e, x_t^e, y_t^e\}_{t=1}^{\infty}$ ,  $\{p_{xt}^e, p_{yt}^e\}_{t=1}^{\infty}$ ,  $\{\mu_t^e\}_{t=1}^{\infty}$  and  $\{\phi_t^e\}_{t=1}^{\infty}$  such that

- 1  $(x_{bt}^e, y_{bt}^e, \hat{y}_t^e)$  solves the banker's problem given  $\phi_t^e$ .
- 2  $(x_{nt}^e, y_{nt}^e)$  solves the non-banker's problem.
- 3  $(x_t^e, y_t^e)$  solves the type 2 agent's problem.
- 4 Markets clear.
- 5 Free entry.
- 6  $\phi_t^e$  solves the bellman equation given  $(x_{bt}^e, y_{bt}^e)$

# Equilibrium

## Analysis of Banker's Problem

### Lemma 4

There is an equilibrium only if  $p_{yt} \leq \rho$ .

### Lemma 5

When  $p_{yt} = \rho$ , there is an equilibrium with no banks.

- No banks means no trade in this setup.

# Equilibrium

## Analysis of Banker's Problem

### Lemma 6

When  $p_{yt} < \rho$ , the repayment constraint must bind,  $\hat{y}_t^e + y_{bt}^e = \phi_t$ .

### Lemma 7

When  $p_{yt} < \rho$ ,  $y_{bt}^e = 0$ .



# Equilibrium

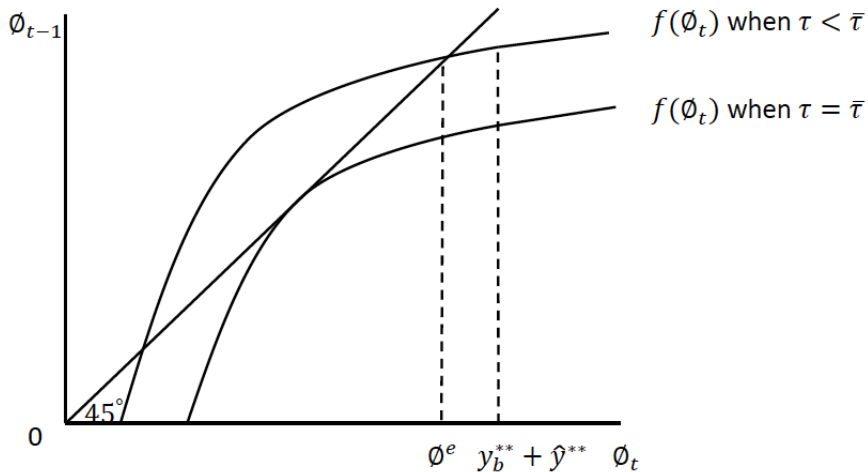
## Analysis of Banker's Problem

### Proposition 3

When  $p_y < \rho$ , if  $0 < \tau \leq \bar{\tau}$ , there are two stationary equilibria with banks (trade), one is stable and the other is unstable;  
if  $\tau = \bar{\tau}$ , there is a unique stationary equilibrium with banks;  
if  $\tau > \bar{\tau}$ , there is an equilibrium with no banks.

# Equilibrium

Steady State in Terms of  $\phi$  When  $p_y < \rho$



## Proposition 4

The competitive equilibrium implements the efficient allocations if the following conditions are satisfied:

(i)  $p_y^e = \frac{\theta}{1-\theta} < \rho$ ,

(ii)  $\tau = \tau^*$ , where  $\tau^*$  solves  $\frac{p_x^e(\tau)x_b^e(\tau)}{\Pi} + \frac{\tau(\rho - \frac{\theta}{1-\theta})}{r\lambda\rho} = k$ ,

(iii)  $t = t^*$ , where  $t^*$  solves

$$u[x_b^e(t)] - u[x_n^e(t)] + \frac{1-\theta}{\theta} \frac{u'[x_n^e(t)][x^e(t) - x_b^e(t)]}{1-\mu^e(t)} = \frac{1-\theta}{\theta} k_0.$$

- The government can also impose a quota by limiting the number of bank charters at the efficiency level  $\mu^*$ .
- A lottery is the easiest way to do the rationing scheme.

### Proposition 5

The competitive equilibrium implements the efficient allocations if the following conditions are satisfied:

(i)  $p_y^e = \frac{\theta}{1-\theta} < \rho$ ,

(ii)  $\mu = \mu^*$ , where  $\mu^*$  is the efficient number of bankers.

# Conclusions

- We develop a model with limited commitment and endogenous monitoring to study the optimal number and size of banks.
- Banking arises endogenously because of economies of scale.
- The planner needs to give the bankers some reward to dissuade opportunistic behavior.
- The optimal number of banks is
  - negatively related to the fixed and marginal monitoring costs,
  - negatively related to the impatience,
  - negatively related to the temptation to default,
  - positively related to the rate of return.

- When the tax on banks is not too high, there exists a non-degenerate stable stationary equilibrium.
- The competitive equilibrium implements the efficient allocations only if the government limits entry of banks. One natural way is to charge a tax on bankers and give a transfer to non-bankers; another way is to simply impose a quota by limiting the number of bank charters.