

The Balassa-Samuelson Hypothesis

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1 Why is it so expensive to live in Japan?

Consider a closed economy with two sectors, tradables and nontradables. Both sectors employ labor and capital as production inputs. The total amount of labor is fixed at $L = L_T + L_N$, while capital is internationally mobile. Tradables can be costlessly transformed to capital, while nontradables cannot. The constant-returns-to-scale (*CRS*) production functions of both the sectors are written as

$$Y_T = A_T F(K_T, L_T), \quad Y_N = A_N G(K_N, L_N)$$

The price of the tradable is normalized to one, while the price of the nontradable is p . Let r be the world interest rate in terms of tradables. w denotes the wage rate.

At each instant of time, firms maximize their profits π_T and π_N that are defined as follows:

$$\begin{aligned} \pi_T &= A_T F(K_T, L_T) - wL_T - rK_T \\ \pi_N &= pA_N G(K_N, L_N) - wL_N - rK_N \end{aligned}$$

Define

$$\begin{aligned} k_T &\equiv \frac{K_T}{L_T}, \quad k_N \equiv \frac{K_N}{L_N}, \\ y_T &= A_T f(k_T) \equiv A_T F(k_T, 1) = \frac{A_T F(K_T, L_T)}{L_T}, \\ y_N &= A_N g(k_N) \equiv A_N G(k_N, 1) = \frac{A_N G(K_N, L_N)}{L_N}. \end{aligned}$$

The first-order conditions for profit maximization are listed as

$$\begin{aligned} A_T f'(k_T) &= r, & A_T [f(k_T) - f'(k_T)k_T] &= w \\ pA_N g'(k_N) &= r, & pA_N [g(k_N) - g'(k_N)k_N] &= w \end{aligned}$$

The zero-profit conditions are written as

$$A_T f(k_T) = rk_T + w, \quad pA_N g(k_N) = rk_N + w.$$

With r constant, taking natural logs of the first equation and differentiating yields

$$\frac{dA_T}{A_T} + \frac{rk_T}{A_T f(k_T)} \frac{dk_T}{k_T} = \frac{rk_T}{A_T f(k_T)} \frac{dk_T}{k_T} + \frac{w}{A_T f(k_T)} \frac{dw}{w}$$

Define

$$\hat{A} = \frac{dA}{A}, \quad \mu_{LT} = \frac{wL_T}{Y_T}, \quad \mu_{LN} = \frac{wL_N}{Y_N}.$$

Note that \hat{X} represents the growth rate (percentage change) of variable X , and μ_{LT} and μ_{LN} represent labor's share in output of tradable and nontradable sectors. Then we can write

$$\hat{A}_T = \mu_{LT} \hat{w}. \tag{1}$$

Similarly, log-differentiation of the zero-profit condition for nontradables yields

$$\hat{p} + \hat{A}_N = \mu_{LN} \hat{w}. \tag{2}$$

Eliminating w from (1) and (2), we obtain

$$\hat{p} = \frac{\mu_{LN}}{\mu_{LT}} \hat{A}_T - \hat{A}_N \tag{3}$$

Assume that $\frac{\mu_{LN}}{\mu_{LT}} > 1$, since the nontradable sector is usually more *labor-intensive* than the tradable sector. Then faster productivity growth in tradables than in nontradables ($\hat{A}_T > \hat{A}_N$) push the price of nontradables upward over time.

The *Balassa-Samuelson* Effect is a tendency for countries with higher productivity in tradables compared with nontradables to have higher price levels.

There are two countries: Home and Foreign. Tradables are priced at one in both Home and Foreign, while nontradables are priced at p and p^* ,

respectively. We assume that the price level is a geometric average, with weights γ and $1 - \gamma$, of the prices of tradables and nontradables. The Home and Foreign price indexes are

$$P = (1)^\gamma p^{1-\gamma} = p^{1-\gamma}, \quad P^* = (1)^\gamma (p^*)^{1-\gamma} = (p^*)^{1-\gamma}$$

Thus the Home-to-Foreign price level ratio is

$$\frac{P}{P^*} = \left(\frac{p}{p^*} \right)^{1-\gamma}$$

Taking logs and differentiating with respect to time, we obtain

$$\hat{P} - \hat{P}^* = (1 - \gamma)(\hat{p} - \hat{p}^*) = (1 - \gamma) \left[\frac{\mu_{LN}}{\mu_{LT}} (\hat{A}_T - \hat{A}_T^*) - (\hat{A}_N - \hat{A}_N^*) \right].$$

where asterisk * denotes variables for the Foreign. Therefore, the Home will experience real appreciation (that is, $\hat{P} - \hat{P}^* > 0$) if its productivity growth advantage in tradables exceeds its productivity growth advantage in non tradables (that is, $(\hat{A}_T - \hat{A}_T^*) > (\hat{A}_N - \hat{A}_N^*)$), provided that nontradables are more labor-intensive than tradables ($\frac{\mu_{LN}}{\mu_{LT}} > 1$).

2 Does productivity growth in the tradable sector increase employment in the nontradable sector?

In this section, we explore the consequence of the productivity growth in the tradable sector on employment, provided that the productivity growth in the nontradable sector equals zero, $\hat{A}_N = 0$.

We assume that the production function is Cobb-Douglas with capital's share equal to α . Then the production function of nontradables is,

$$Y_N = A_N K_N^\alpha L_N^{1-\alpha} = A_N k_N^\alpha L_N.$$

Taking logs and differentiating, we have

$$\hat{L}_N = \hat{Y}_N - \alpha \hat{k}_N = \hat{C}_N - \alpha \hat{k}_N, \quad (4)$$

where C represents consumption, and the last inequality follows since domestic demand equals supply are equal in nontradables. That is, the employment growth in the nontradable sector is increasing in its demand growth and decreasing in the growth of its capital-labor ratio.

2.1 Calculating \hat{C}_N

At each instant, the representative consumer maximizes a CES utility function;

$$\left[\gamma^{\frac{1}{\theta}} C_T^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} C_N^{\frac{\theta-1}{\theta}} \right], \quad \gamma \in (0, 1), \theta > 0 \quad (5)$$

where θ is the elasticity of substitution between tradables and nontradables. As θ approaches 1, the utility function becomes proportional to the Cobb-Douglas function, $C_T^\gamma C_N^{1-\gamma}$. The budget constraint is written as

$$C_T + pC_N = Z \equiv w(r)L + r\bar{Q} \quad (6)$$

where \bar{Q} is the financial wealth held at this each instant of time. Maximizing (5) subject to (6) yields

$$\frac{pC_N}{C_T} = \left(\frac{1-\gamma}{\gamma} \right) p^{1-\theta} \quad (7)$$

Note that the ratio of consumption of nontradables to tradables $\frac{pC_N}{C_T}$ becomes $\frac{1-\gamma}{\gamma}$ as θ approaches 1 and the utility function becomes Cobb-Douglas.

Combining (7) with (6), we obtain the demand functions for tradables and nontradables,

$$C_T = \frac{\gamma Z}{\gamma + (1-\gamma)p^{1-\theta}}, \quad C_N = \frac{p^{-\theta}(1-\gamma)Z}{\gamma + (1-\gamma)p^{1-\theta}}$$

Taking logs of C_N and differentiating, assuming $p = 1$ initially, we have

$$\hat{C}_N = \hat{Z} - (\gamma\theta + 1 - \gamma)\hat{p} \quad (8)$$

From the budget constraint (6), we have

$$\hat{Z} = \frac{w(r)L}{w(r)L + r\bar{Q}} \hat{w} \equiv \psi_L \hat{w} = \frac{\psi_L}{\mu_{LT}} \hat{A}_T \quad (9)$$

where ψ_L is the share of labor income in total GNP at the initial steady state, and the last equality follows (1). Finally, (3) with $\hat{A}_N = 0$ yields the growth rate of the price of nontradables as

$$\hat{p} = \frac{1-\alpha}{\mu_{LT}} \hat{A}_T. \quad (10)$$

Substituting (9) and (10) into (8), we obtain

$$\hat{C}_N = [\psi_L - (1-\alpha)(\gamma\theta + 1 - \gamma)] \frac{\hat{A}_T}{\mu_{LT}}. \quad (11)$$

2.2 Calculating \hat{k}_N

The remaining task is to find $\alpha\hat{k}_N$ in terms of \hat{A}_T . The profit maximization of the firm in the nontradable sector implies

$$pA_N\alpha k_N^{\alpha-1} = r.$$

Taking logs and differentiating yields $\hat{p} + (\alpha - 1)\hat{k}_N = 0$. Then

$$\alpha\hat{k}_N = \frac{\alpha}{1-\alpha}\hat{p} = \frac{\alpha}{1-\alpha}\frac{1-\alpha}{\mu_{LT}}\hat{A}_T = \frac{\alpha}{\mu_{LT}}\hat{A}_T \quad (12)$$

Substituting (11) and (12) into (4), we have

$$\hat{L}_N = [\psi_L - (1-\alpha)(\gamma\theta + 1 - \gamma) - \alpha]\frac{\hat{A}_T}{\mu_{LT}}.$$

Observe that the sign of \hat{L}_N is ambiguous. If the utility function is Cobb-Douglas (that is, $\theta = 1$),

$$\hat{L}_N = (\psi_L - 1)\frac{\hat{A}_T}{\mu_{LT}}$$

Since ψ_L , labor share in GNP, is always below 1, $\hat{L}_N < 0$. That is, the employment in the nontradable sector goes down as productivity grows in the tradable sector. However, constant expenditure share is not an widely observed fact in the real world economy. The share of consumption of nontradable has been rising in the developed countries in the last several decades. If θ is sufficiently below 1, $\hat{L}_N > 0$, and the employment share of nontradables rises as productivity grows in the tradable sector.