

An Economic Analysis of the Covid-19 Pandemic with Explicit Medical Capacity Constraint

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The Fujita-Nakata Model (2021)

Transition:

$$\text{Susceptible: } S(t+1) = S(t)[1 - \delta_t] - T(t)$$

$$\text{Infected : } I(t+1) = I(t) + T(t) - (\pi_r + \pi_d)I(t)$$

$$\text{Recovered: } R(t+1) = R(t) + \pi_r I(t) + \delta_t S(t)$$

$$\text{Dead: } D(t+1) = D(t) + \pi_d I(t)$$

$$\text{Vaccination: } V_{t+1} = V_t + \delta_t S(t)$$

$\delta(t)$: probability to be vaccinated

$$\delta_t = \min \left\{ \frac{M}{S(t)}, 1 \right\}$$

where M is the vaccination capacity offered by the government

$T(t)$: mass of newly infected agents

π_r : probability to be recovered

π_d : probability to die

Output:

$$Y_t = (1 - \alpha_t)A_t(\{\alpha_\tau\}_{\tau=0}^{t-1})(S_t + I_t + R_t)$$

where α_t denotes action to avoid infection. The productivity depends on the past actions $\{\alpha_\tau\}_{\tau=0}^{t-1}$.

Newly Infected mass:

$$T(t) = \frac{\beta_t(1 - h\alpha_t)^k S_t I_t}{S_t + I_t + R_t}$$

The Kubota Model

A version of Eichenbaum et al. (2020a).

Time is discrete. There are two sectors.

People are infected by the virus if they consume a unit of service good of sector 1.

They are not infected by consuming a unit of good of sector 2 through online shopping.

People work at sector 1 or sector 2. They are not infected by the virus at job places.

“Soft lockdown” is captured by the consumption tax at rate μ , which is charged on sector 1.

Each agent maximizes the following discounted sum of utilities:

$$\sum_{t=0}^{\infty} \beta^t \left[\ln c_t - \theta \frac{n_t^2}{2} \right]$$

where n_t is hours of work, and c_t is the consumption defined as

$$c_t = \left[\frac{1}{2} c_{1t}^{1-1/\eta} + \frac{1}{2} c_{2t}^{1-1/\eta} \right]$$

where η is the elasticity of substitution, which captures the substitution of Good 1 for Good 2.

The production function is

$$y_{it} = A_{it} n_{it}, \quad i = 1, 2$$

Transition

$$\begin{aligned}\text{Susceptible:} & \quad S(t+1) = S(t)[1 - \delta_t] - T(t) \\ \text{Infected} & \quad : \quad I(t+1) = I(t) + T(t) - (\pi_r + \pi_d)I(t) \\ \text{Recovered:} & \quad R(t+1) = R(t) + \pi_r I(t) + \delta_t S(t) \\ \text{Dead:} & \quad D(t+1) = D(t) + \pi_d I(t) \\ \text{Vaccination} & \quad : \quad V_{t+1} = V_t + \delta_t S(t)\end{aligned}$$

$\delta(t)$: probability to be vaccinated

$$\delta_t = \min \left\{ \frac{M}{S(t)}, 1 \right\}$$

where M is the vaccination capacity offered by the government

$T(t)$: mass of newly infected agents

π_r : probability to be recovered

π_d : probability to die

Effective Reproduction # and Perception to Infection Probability

mass of newly infected: $T(t) = \pi_c S(t) c_{1t}^S * I(t) c_{1t}^I$

probability for a susceptible agent to be infected:

$$\tau_t(c_{1t}^S) = \frac{T(t)}{S(t)} = \pi_c c_{1t}^S * I(t) c_{1t}^I$$

The effective reproduction number is defined as:

$$R_t = \frac{T_t}{(\pi_r + \pi_d)[S(t) + I(t) + R(t)]I(t)}$$

A susceptible person's perception rate:

$$\psi_{t+1} = \psi_t - \varphi \frac{(1 - \psi_t)\psi_t^2}{\psi_0}$$

where ψ_0 is the initial level of perception rate and φ is the speed adjustment parameter.

Optimization: Susceptible

The value function for a susceptible person

$$U_t^S = \frac{\eta}{\eta - 1} \ln \left[\frac{1}{2} (c_{1t}^S)^{1-1/\eta} + \frac{1}{2} (c_{2t}^S)^{1-1/\eta} \right] - \theta \frac{(n_t^S)^2}{2} \\ + \beta \{ \psi_t \tau_t (c_{1t}^S) U_{t+1}^I + \delta_t U_{t+1}^R + [1 - \psi_t \tau_t (c_{1t}^S) - \delta_t] U_{t+1}^S \}$$

The budget constraint

$$(1 + \mu_t) c_{1t}^S + c_{2t}^S = A n_t^S$$

Solution:

$$\frac{(c_{1t}^S)^{1-1/\eta}}{(c_{1t}^S)^{1-1/\eta} + (c_{2t}^S)^{1-1/\eta}} = (1 + \mu_t) \left(\frac{\theta}{A} \right) n_t^S + \beta \psi_t \pi_c l(t) c_{1t}^l [U_{t+1}^S - U'_{t+1}]$$

$$\frac{(c_{2t}^S)^{1-1/\eta}}{(c_{1t}^S)^{1-1/\eta} + (c_{2t}^S)^{1-1/\eta}} = \left(\frac{\theta}{A} \right) n_t^S$$

We can obtain a closed solution for $(c_{1t}^S, c_{2t}^S, n_t^S)$.

The value function for an infected person, who will not be reinfected.

$$U_t^I = \frac{\eta}{\eta - 1} \ln \left[\frac{1}{2} (c_{1t}^I)^{1-1/\eta} + \frac{1}{2} (c_{2t}^I)^{1-1/\eta} \right] - \theta \frac{(n_t^I)^2}{2} + \beta \{ \pi_r U_{t+1}^R + \pi_d \times 0 + [1 - \pi_r - \pi_d] U_{t+1}^I \}$$

The budget constraint

$$(1 + \mu_t) c_{1t}^I + c_{2t}^I = A n_t^I$$

Solution:

$$\frac{(c_{1t}^I)^{1-1/\eta}}{(c_{1t}^I)^{1-1/\eta} + (c_{2t}^I)^{1-1/\eta}} = (1 + \mu_t) \left(\frac{\theta}{A}\right) n_t^I + \beta \psi_t \pi_c I(t) c_{1t}^I [U_{t+1}^S - U_{t+1}^I]$$

$$\frac{(c_{2t}^I)^{1-1/\eta}}{(c_{1t}^I)^{1-1/\eta} + (c_{2t}^I)^{1-1/\eta}} = \left(\frac{\theta}{A}\right) n_t^I$$

We can obtain a closed solution for $(c_{1t}^I, c_{2t}^I, n_t^I)$.

The value function for a recovered person, who will not be reinfected.

$$U_t^R = \frac{\eta}{\eta - 1} \ln \left[\frac{1}{2} \left(c_{1t}^R \right)^{1-1/\eta} + \frac{1}{2} \left(c_{2t}^R \right)^{1-1/\eta} \right] - \theta \frac{(n_t^R)^2}{2} + \beta U_{t+1}^R$$

The budget constraint

$$(1 + \mu_t) c_{1t}^R + c_{2t}^R = A n_t^R$$

Solution: The allocation of a recovered person is the same as that of an infected person. See Krueger et al (2020).

Hospitalization and Immunodefficiency

Susceptible: $S(t+1) = S(t)[1 - \delta_t] - T(t)$

Infected : $I(t+1) = I(t) + (1 - h_t)T(t) - (\pi_r + \pi_d)I(t)$

hospitalized: $H(t+1) = H(t) + h_t T(t) - (q_r + q_d)H(t)$

capacity: $Q(t+1) = \max\{Q(t) - h_t T(t) + (q_r + q_d)H(t), 0\}$

Recovered: $R(t+1) = R(t) + \pi_r I(t) + \delta_t S(t) - gR(t)$

Dead: $D(t+1) = D(t) + \pi_d I(t) + q_d H(t)$

Vaccination: $V_{t+1} = V_t + \delta_t S(t)$

where h_t is the probability that one can be hospitalized

$$\begin{aligned} h_t &= 1, \text{ if } T(t) \leq Q(t) \\ &= \frac{Q(t)}{T(t)}, \text{ if } T(t) > Q(t) \end{aligned}$$

g : probability of losing immunity

q_r : probability to be recovered in the hospital

q_d : probability to die in the hospital

The value function for a susceptible person

$$\begin{aligned} U_t^S &= \frac{\eta}{\eta - 1} \ln \left[\frac{1}{2} (c_{1t}^S)^{1-1/\eta} + \frac{1}{2} (c_{2t}^S)^{1-1/\eta} \right] - \theta \frac{(n_t^S)^2}{2} \\ &\quad + \beta \{ \psi_t \tau_t (c_{1t}^S) (1 - h_t) U_{t+1}' + \psi_t \tau_t (c_{1t}^S) h_t U_{t+1}' + \delta_t U_{t+1}^R \\ &\quad + [1 - \psi_t \tau_t (c_{1t}^S) - \delta_t] U_{t+1}^S \} \end{aligned}$$

The budget constraint

$$(1 + \mu_t) c_{1t}^S + c_{2t}^S = A n_t^S$$

The value function for an infected person, who will not be reinfected.

$$U_t^I = \frac{\eta}{\eta - 1} \ln \left[\frac{1}{2} (c_{1t}^I)^{1-1/\eta} + \frac{1}{2} (c_{2t}^I)^{1-1/\eta} \right] - \theta \frac{(n_t^I)^2}{2} + \beta \{ \pi_r U_{t+1}^R + \pi_d \times 0 + [1 - \pi_r - \pi_d] U_{t+1}^I \}$$

The budget constraint

$$(1 + \mu_t) c_{1t}^I + c_{2t}^I = A n_t^I$$

Optimization: Hospitalized

The value function for an Hospitalized person, who will not be reinfected.

$$U_t^H = \frac{\eta}{\eta - 1} \ln \left[\frac{1}{2} (c_{1t}^H)^{1-1/\eta} + \frac{1}{2} (c_{2t}^H)^{1-1/\eta} \right] \\ + \beta \{ q_r U_{t+1}^R + q_d \times 0 + [1 - q_r - q_d] U_{t+1}^H \}$$

The budget constraint

$$(1 + \mu_t) c_{1t}^H + c_{2t}^H = w_t \\ c_{1t}^H = 0$$

The value function for a recovered person, who will not be reinfected.

$$U_t^R = \frac{\eta}{\eta - 1} \ln \left[\frac{1}{2} (c_{1t}^R)^{1-1/\eta} + \frac{1}{2} (c_{2t}^R)^{1-1/\eta} \right] - \theta \frac{(n_t^R)^2}{2} + \beta g U_{t+1}^S + \beta (1 - g) U_{t+1}^R$$

The budget constraint

$$(1 + \mu_t) c_{1t}^R + c_{2t}^R = A n_t^R$$

no optimization

An Economy Model in Discrete Time

Healthy:

$$N_H(t+1) = N_H(t) - \frac{\alpha N_H(t) N_A(t)}{2} d$$

Infected and don't know:

$$N_A(t+1) = \frac{\alpha N_H(t) N_A(t)}{2} d(1-s) + \mu N_G(t) - (s+f) N_A(t) + N_A(t)$$

Diagnosed positive and light:

$$N_S(t+1) = \frac{\alpha N_H(t) N_A(t)}{2} ds(1-e) + s(1-e) N_A(t) - (e+f) N_S(t) + N_S(t)$$

Diagnosed positive and medically treated:

$$N_T(t+1) = se N_A(t) + e N_S(t) - (k+\lambda) N_T(t) + N_T(t)$$

Recovered and Immune:

$$N_G(t+1) = \lambda N_T(t) + f[N_S(t) + N_A(t)] - \mu N_G(t) + N_G(t)$$

Basic Reproduction Number 1

In the original SIR model,

$$\frac{dI(t)}{dt} = [\beta S(t) - \gamma]I(t)$$

At time 0, the virus spreads if and only if $\beta S(0) - \gamma > 0$, or equivalently,

$$R_0 = \frac{\beta S(0)}{\gamma} > 1$$

where R_0 is called the "Basic Reproduction Number".

Let p be the "Herd Immunity Threshold", which is defined by

$R_0(1 - p) = 1$, or equivalently,

$$p = 1 - \frac{1}{R_0} = \begin{cases} .33, & \text{for } R_0 = 1.5 \\ .50, & \text{for } R_0 = 2 \\ .66, & \text{for } R_0 = 3 \end{cases}$$

Basic Reproduction Number 2

In my model, let $A(t) = N_A(t) + N_S(t) + N_T(t)$. $A(t)$ denotes the mass of infected people. We can write

$$A(t+1) - A(t) = N_A(t) \left[\frac{d\alpha(t)N_H(t) - 2f}{2} \right] + \mu N_G(t) - fN_S(t) - (k + \lambda)A(t)$$

Specifically, at time 0, $N_H(0) = 1$, $N_G(0) = N_S(0) = N_T(0) = 0$. Then

$$A(1) - A(0) = N_A(0) \left[\frac{d\alpha(0) - 2f}{2} \right]$$

Therefore, the basic reproduction number is given by

$$R_0 = \frac{d\alpha(0)}{2f}$$

With $d = .2$, $f = .7$, we obtain

$$\begin{aligned} R_0 &= 2.286, \text{ for } \alpha(0) = 16 \\ &= 1.429, \text{ for } \alpha(0) = 10 \end{aligned}$$

Notations

Basic Idea: Two persons meet and produce economic output, and transfer virus.

Labor Force: $L = N_H + N_A + N_G$, Infected: $S = N_A + N_S + N_T$

and

- α : meeting rate, d : infection rate
- s : rate of being diagnosed positive ($1 - s$: negative)
- f : recovery rate
- e : rate of being heavily ill ($1 - e$: lightly ill)
- k : death rate
- μ : rate of losing antibody (immunodeficient)

The Output

In the service economy, output is produced by contacts by people.

	N_H	N_A	N_G
N_H	y (counted twice)	y	y
N_A	y	y (counted twice)	y
N_G	y	y	y (counted twice)

Let N denote # of people.

There are combinations as many as

$${}_N C_2 = \frac{N(N-1)}{2} \simeq \frac{N^2}{2}, \text{ as } N \text{ is large enough}$$

The appropriate measure for aggregate output is

$$y_t = \frac{\alpha_t (N_H^2 + N_A^2 + N_G^2 + 2N_H N_A + 2N_A N_G + 2N_H N_G)}{2}$$

Baseline Parameters

The baseline parameters are given as follows.

$$\begin{aligned}\rho &= 0, \rho_1 = .1, \rho_2 = .2, \\ \eta &= .1, s = .2, \sigma = 1, \lambda = .4, \mu = 0 \\ d &= .2, e = .3, f = .7, k = .05 \\ \alpha_0 &= 16, \alpha_G = 10\end{aligned}$$

where α_0 is the initial level of meeting rate, while α_G is the target level of meeting rate that is set by the government.

Simulation Scenarios

- Changing death rate (k) does not affect the results so much.
- Any positive losing antibody rate (μ) might destroy human beings.
- The COVID-19 might be even more threatening than HIV, since the former's infection rate is much higher than the latter.
- Simulation has too many scenarios, partly because government takes many prevention measures in the real world.
- Without prevention measures, the final deaths are projected to be as much as 400,000 (comparable to the prediction by the expert community).

Baseline Setting

Consider exercises that government reduces the meeting rate from $\alpha_0 = 16$ to $\alpha_G = 10$ with different speeds.

$$\alpha_{t+1} = \alpha_t(1 - \rho) + \rho\alpha_G$$

where $\rho = .1$ or $\rho = .2$ represents the speed of adjustment. The higher ρ is, the faster the reduction is.

- Nt stands for the serious patients to be treated at hospitals.
- c stands for the medical capacity ($c = 0.001$ equals 100 thousands for population 100 millions).
- Beds as much as 100 thousands are not enough to accept patients at the pandemic peak. Beds of only less than 20 thousands are available in Japan.
- Numerical exercises suggest that "The earlier reduction in the contact rate (α) is more effective than the lower target (α_G)."
- The government should take whatever containment policies as early as possible

Lockdown-recovery-lockdown

Consider a lockdown-recovery-lockdown policy such that

$$\alpha_{t+1} = (1 - \rho)\alpha_t + \rho\alpha_G, \text{ for } t < 30$$

$$\alpha_{t+1} = (1 - \eta)\alpha_t + \eta\alpha_0, \text{ for } 30 \leq t < 60$$

$$\alpha_{t+1} = (1 - \rho)\alpha_t + \rho\alpha_G, \text{ for } t \geq 60$$

where

$$\alpha_0 = 16: \text{ initial meeting rate}$$

$$\alpha_G = 10: \text{ meeting rate target}$$

and

$$\rho : \text{ reduction rate}$$

$$\eta = .1: \text{ recovery rate}$$

The Spanish Flu 1918-1919

Lockdown-recovery-lockdown-recovery

A lockdown-recovery-lockdown-recovery process like the Spanish Flu is simulated. The meeting rate changes as follows.

$$\alpha_{t+1} = (1 - \rho)\alpha_t + \rho\alpha_G, \text{ for } t < 30$$

$$\alpha_{t+1} = (1 - \eta)\alpha_t + \eta\alpha_0, \text{ for } 30 \leq t < 60$$

$$\alpha_{t+1} = (1 - \rho)\alpha_t + \rho\alpha_G, \text{ for } t \geq 60$$

$$\alpha_{t+1} = (1 - \eta)\alpha_t + \eta\alpha_0, \text{ for } 60 \leq t < 90$$

$$\alpha_{t+1} = (1 - \rho)\alpha_t + \rho\alpha_G, \text{ for } t \geq 120$$

where $\rho = 0, .1, .2$ is the parameter representing the speed to the lockdown target, while $\eta = .1$ is the speed of recovery to the initial meeting rate.

$\rho = 0$ is the case that there is no contact control, while the target meeting rate is reached faster for $\rho = .2$ than for $\rho = .1$

The Susceptible Agents

Let $V_H, V_A, V_S, V_T, V_Z, V_G$ be the value functions of the states. They satisfy the Bellman equations as follows.

$$V_H(t) = \left(1 - \frac{\alpha N_A(t)}{2}\right) \beta V_H(t+1): \text{meet whom I can work with}$$
$$+ \frac{\alpha N_A(t) + \alpha N_H(t) + \alpha N_G(t)}{2} u(y): \text{meet whom I can work with}$$
$$+ \frac{\alpha N_A(t)}{2} \beta \left[\begin{array}{l} (1-d)V_H(t+1): \text{not infected} \\ +d(1-s)V_A(t+1): \text{infected but not recognized} \\ +ds(1-e)V_S(t+1): \text{infected and not so serious} \\ +dse(1-z)V_T(t+1): \text{infected and cared at a hospital} \\ +dsezV_Z(t+1): \text{infected and cared at home} \end{array} \right]$$

The Infectious Agents

$$V_A(t) = \frac{\alpha N_A(t) + \alpha N_H(t) + \alpha N_G(t)}{2} u(y): \quad \text{meet whom I}$$
$$+ \beta [1 - s(1 - e) - zse - (1 - z)se - f] V_A(t + 1): \text{ no change occurs}$$
$$+ \beta \left[\begin{array}{l} s(1 - e) V_S(t + 1): \text{ showing light symptoms} \\ + (1 - z)se V_T(t + 1): \text{ showing serious symptoms and going to a hosp} \\ + zse V_Z(t + 1): \text{ showing serious symptoms and cared at a hospital} \\ + fV_G(t + 1): \text{ recovered to be immune} \end{array} \right]$$

The Other Values

$$\begin{aligned}V_S(t) &= [1 - (1 - z)e - ze - f]\beta V_S(t + 1) \\ &\quad + \beta[(1 - z)eV_T(t + 1) + zeV_Z(t + 1) + fV_G(t + 1)] \\ V_T(t) &= \beta[1 - \lambda - k]V_T(t + 1) + \beta[\lambda V_G(t + 1) + kV(D)] \\ V_Z(t) &= (1 - \psi - l)V_Z(t + 1) + \beta[\psi V_G(t + 1) + lV(D)] \\ V_G(t) &= \frac{\alpha N_A(t) + \alpha N_H(t) + \alpha N_G(t)}{2}u(y) + \beta[(1 - \mu)V_G(t + 1) + \mu V_A]\end{aligned}$$

People act forward-looking. Then the value functions are solved by backward induction. Solving value functions is equivalent to solving a terminal value problem, while the distribution of people is solved as a usual initial-value problem.

The Steady State Value

The economy reaches a steady state at time T at which

$$N_A(t) = 0, \quad N_H(t) + N_G(t) < 1, \quad \text{for } t \geq T$$

with the steady state value functions satisfying

$$V_H(T) = \frac{\alpha N_H(T) + \alpha N_G(T)}{2} u(y) + \beta V_H(T)$$

$$V_A(t) = \frac{\alpha N_H(T) + \alpha N_G(T)}{2} u(y) \\ + \beta [1 - s(1 - e) - zse - (1 - z)se - f] V_A(T) \\ + \beta [s(1 - e)V_S(T) + (1 - z)seV_T(T) + zseV_Z(T) + fV_G(T)]$$

$$V_G(T) = \frac{\alpha N_H(T) + \alpha N_G(T)}{2} u(y) + \beta [(1 - \mu)V_G(T) + \mu V_A(T)]$$

$$V_S = \frac{\beta [(1 - z)eV_T + zeV_Z + fV_G]}{(1 - z)e + ze + f}, \quad V_T = \frac{\beta [\lambda V_G + kV(D)]}{\lambda + k}, \quad V_Z = \frac{\beta [\psi V_G]}{\psi + \lambda}$$

Terminal Condition

The terminal values at time T are solved as

$$\begin{array}{rcl} Vh_T & & 158.72 \\ Va_T & & 154.15 \\ Vs_T & & 141.54 \\ Vt_T & = & 126.31 \\ Vz_T & & 90.02 \\ Vg_T & & 158.72 \end{array}$$

where Vz_T denotes the value of being ill without being hospitalized.
The file: corona047-value-dynamics-correct.







Summary 1

- 1 There is a trade-off: Stronger lockdown policy restricts the mass of people infected within medical capacity in the first strike, but immunity remains at a relatively low level, and the community is vulnerable to the upcoming virus strike.
- 2 A higher level of initial immunity (like BCG) reduces the required immunity level necessary to stop pandemic.
- 3 Repeated containment policy might be effective to reduce intertemporal loss of lives and outputs by raising the initial level of immunity at each virus strike.
- 4 More intensive testing shortens the duration of the first strike, but makes the community more vulnerable to the second strike.

Summary 2

- ① After all, no or less mobility control will result in more long-term workforce loss, but in smaller short-term economic loss.
- ② In order to evaluate economic welfare, the question is how to take the lost utility of dead people into account.
- ③ The total utility goes down as the population declines.
- ④ Can we evaluate economic welfare appropriately, by simply adding the lifetime utility of people alive?

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