

Cost Functions and Supply

Varian: Ch.20 - 23

Cost Minimization

The firm's cost minimization problem is :

$$\min w_1x_1 + w_2x_2$$

$$\text{such that } f(x_1, x_2) = y$$

The isocost curve is a pair (x_1, x_2) that gives the same cost, C :

$$C(x_1, x_2) = w_1x_1 + w_2x_2, \text{ or } x_2 = \frac{C - w_1x_1}{w_2} = -\frac{w_1}{w_2}x_1 + \frac{C}{w_2}$$

The slope of the isocost is given as $-\frac{w_1}{w_2}$

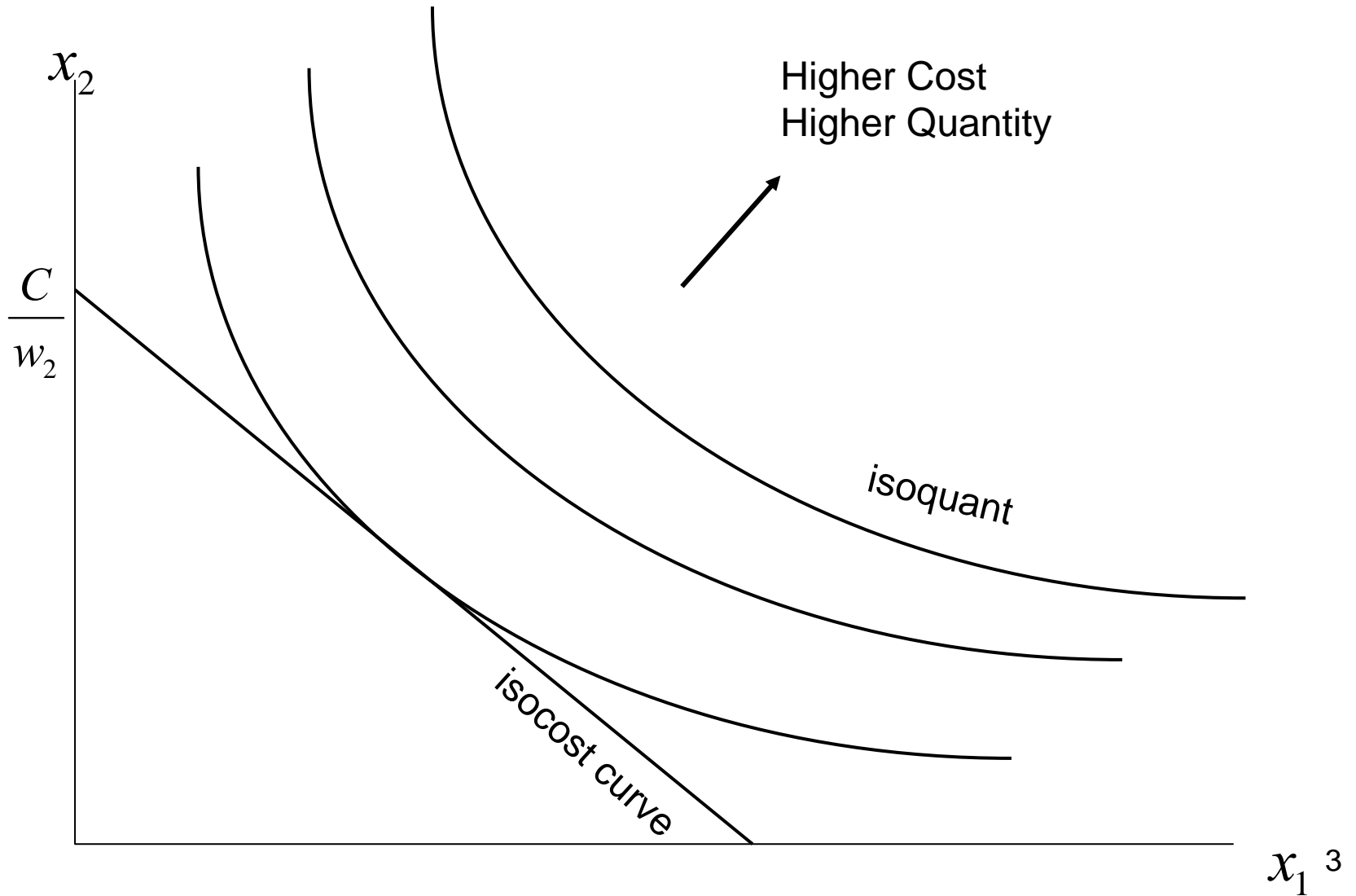
The isoquant is a pair (x_1, x_2) that gives the same output, y :

$$y = f(x_1, x_2)$$

The slope of the isocost is given as

$$\frac{dx_2}{dx_1} = -\frac{\partial f(x_1, x_2) / \partial x_1}{\partial f(x_1, x_2) / \partial x_2} \equiv -\frac{f_1}{f_2}$$

Graph of Cost Minimization



The solution of cost minimization satisfies

$$\frac{f_1}{f_2} = \frac{w_1}{w_2}$$

which says that

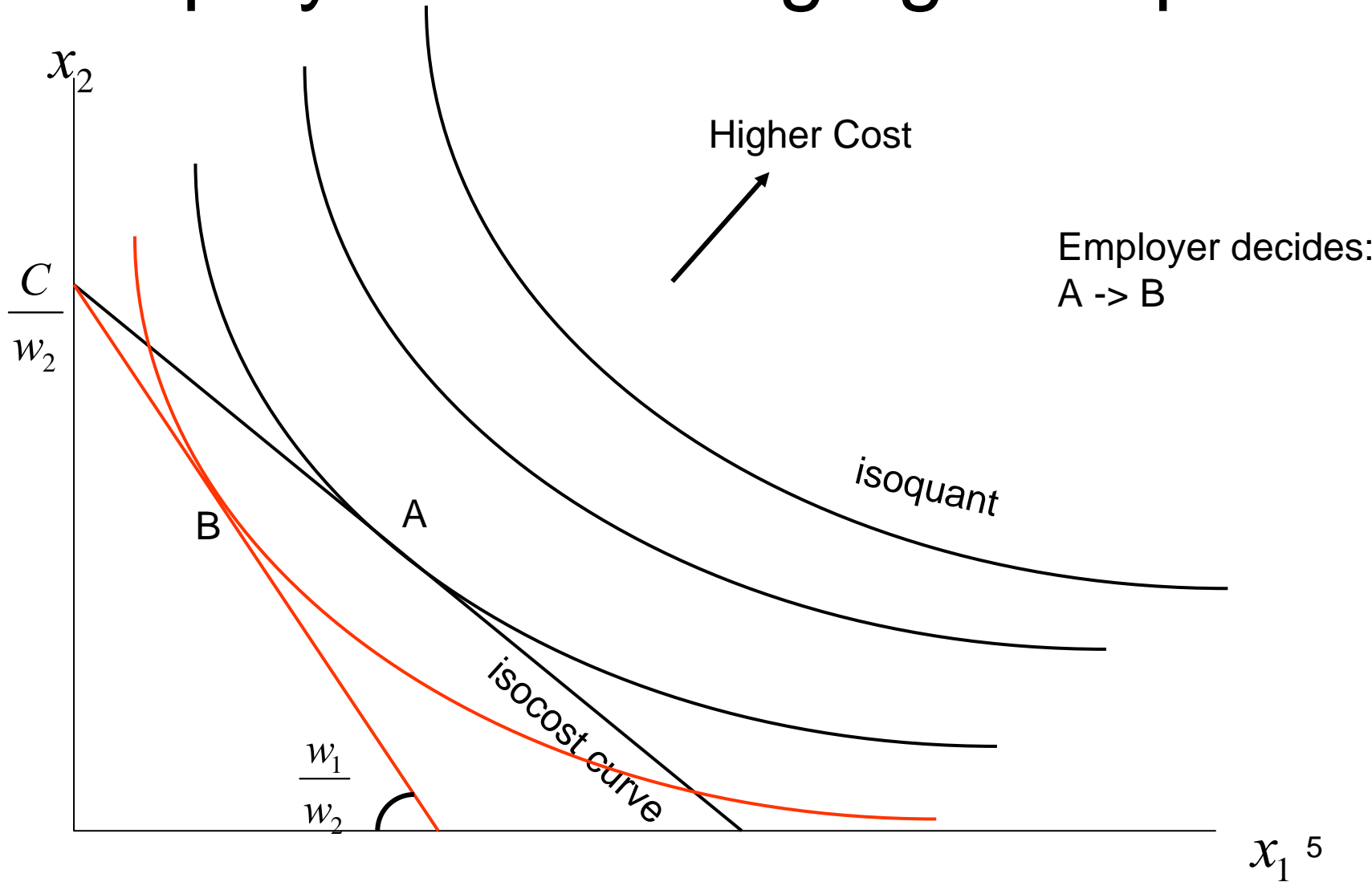
rate of marginal transformation (MRT) = ratio of factor prices

We usually assume that the MRT is decreasing in x_1 .

Then a rise in w_1 reduces input of factor 1 (x_1).

For example, if the wage goes up, your employer will reduce employment, and replaces workers with capital (machines).

Your employer reduces employment as wage goes up.



Short-Run and Long-Run Costs

The firm's cost minimization problem is :

$$\min_{(x_1, x_2)} w_1 x_1 + w_2 x_2$$

$$\text{such that } f(x_1, x_2) = y$$

Assume that the firm cannot change x_2 in the short run.

Then the short - run cost minimization problem is

to minimize $C(x_1, x_2)$ taking x_2 as given :

$$\min_{x_1} w_1 x_1 + w_2 x_2$$

$$\text{such that } f(x_1, x_2) = y$$

For example, an automobile company can change employment but the plant size in the short run. But in the long run, it changes the plant size as well.

Sunk Costs

- Sunk costs are fixed costs that are not recoverable once the production facility is installed.
- The cost for painting the office is a sunk cost, while the cost of buying the furniture is a fixed cost but a sunk cost.
- The cost of buying airplanes is a huge fixed cost for airline industry, but it is not sunk.

The cost function

A cost function is a function of the output y after the inputs are chosen by the producer to minimize the production cost.

There are

total cost : $c(y) = c_v(y) + F$.

average variable cost : $AVC(y) = \frac{c_v(y)}{y}$.

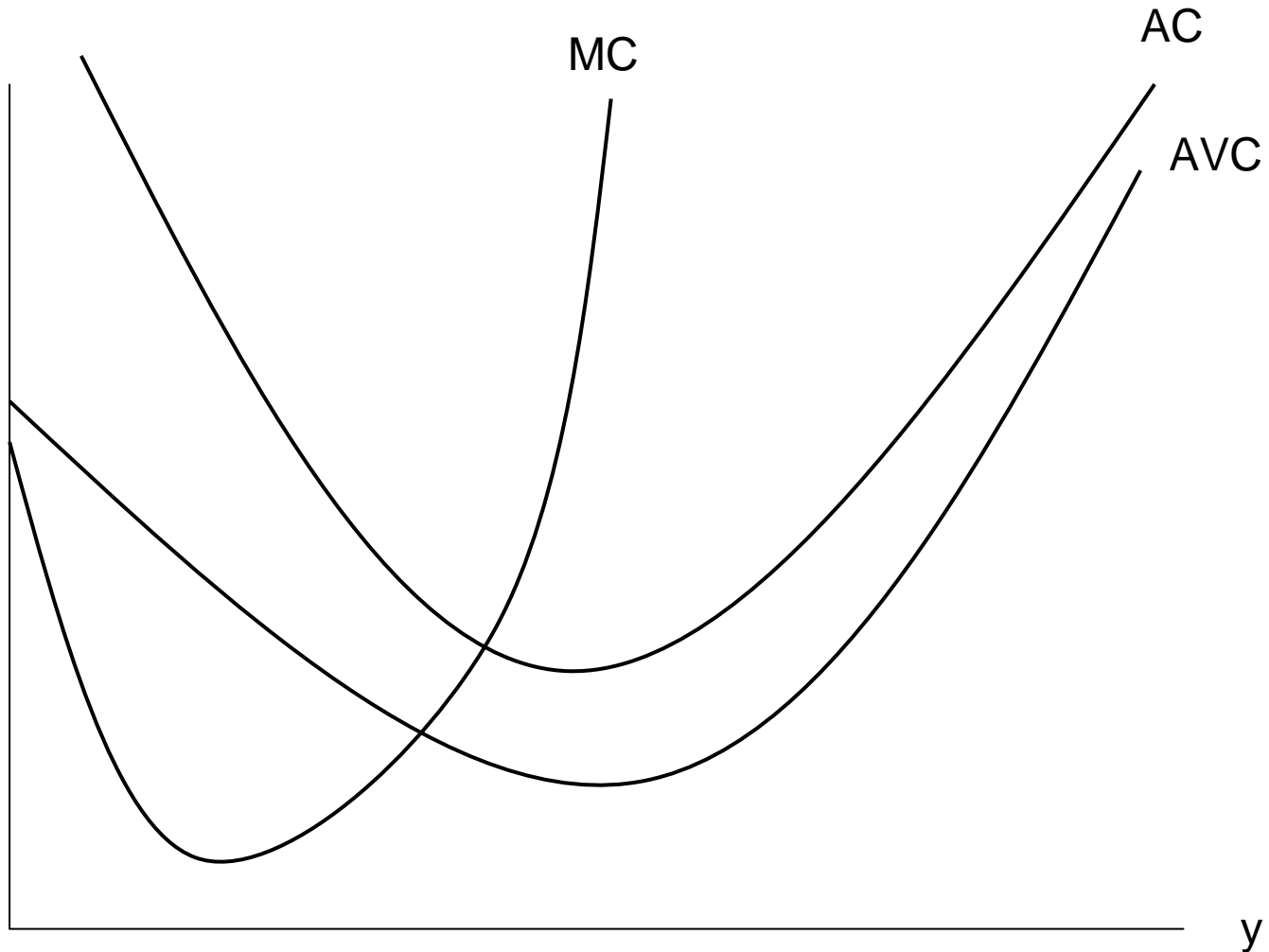
average fixed cost : $AFC(y) = \frac{F}{y}$.

average cost = average variable cost + average fixed cost :

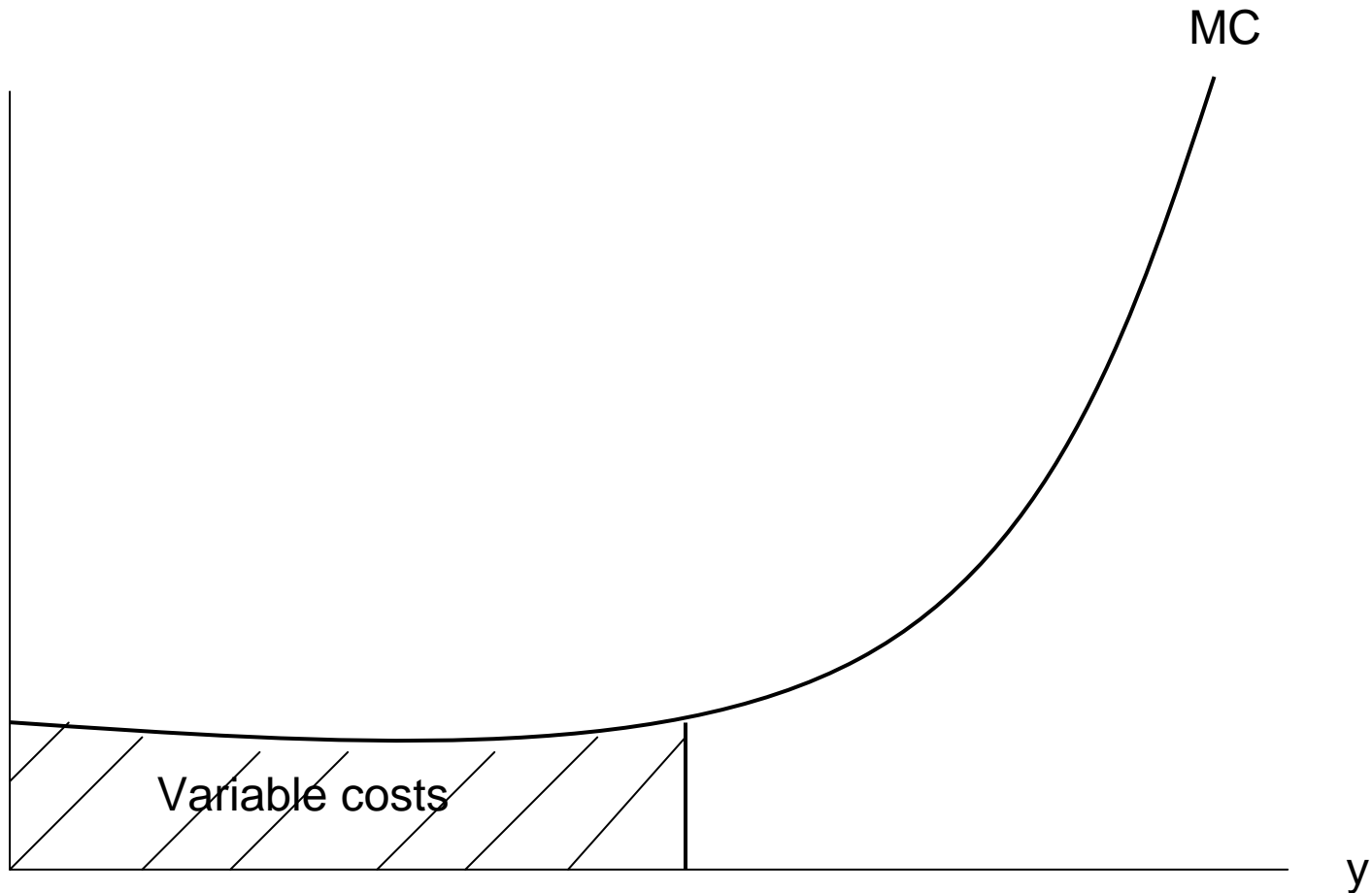
$$AC(y) = \frac{c(y)}{y} = \frac{c_v(y)}{y} + \frac{F}{y},$$

marginal cost : $MC(y) = \frac{dc(y)}{dy} = \frac{dc_v(y)}{dy}$.

Cost Curves



Marginal costs and variable costs



Long run and short run

The short - run cost is no less than the long - run cost.

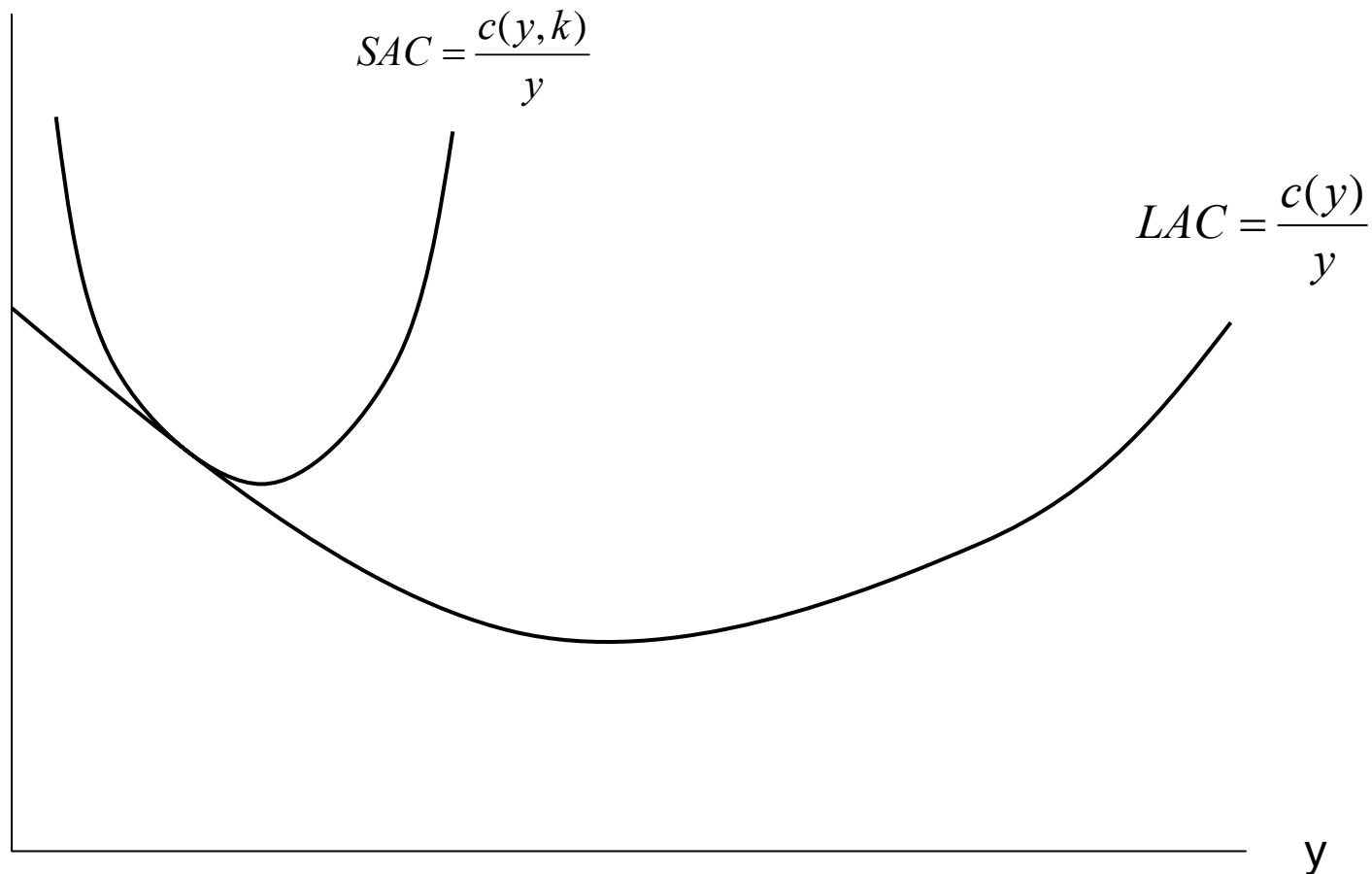
Then we can say that the same thing for the average cost.

$$c(y) \leq c(y, k) \Rightarrow \frac{c(y)}{y} \leq \frac{c(y, k)}{y}$$

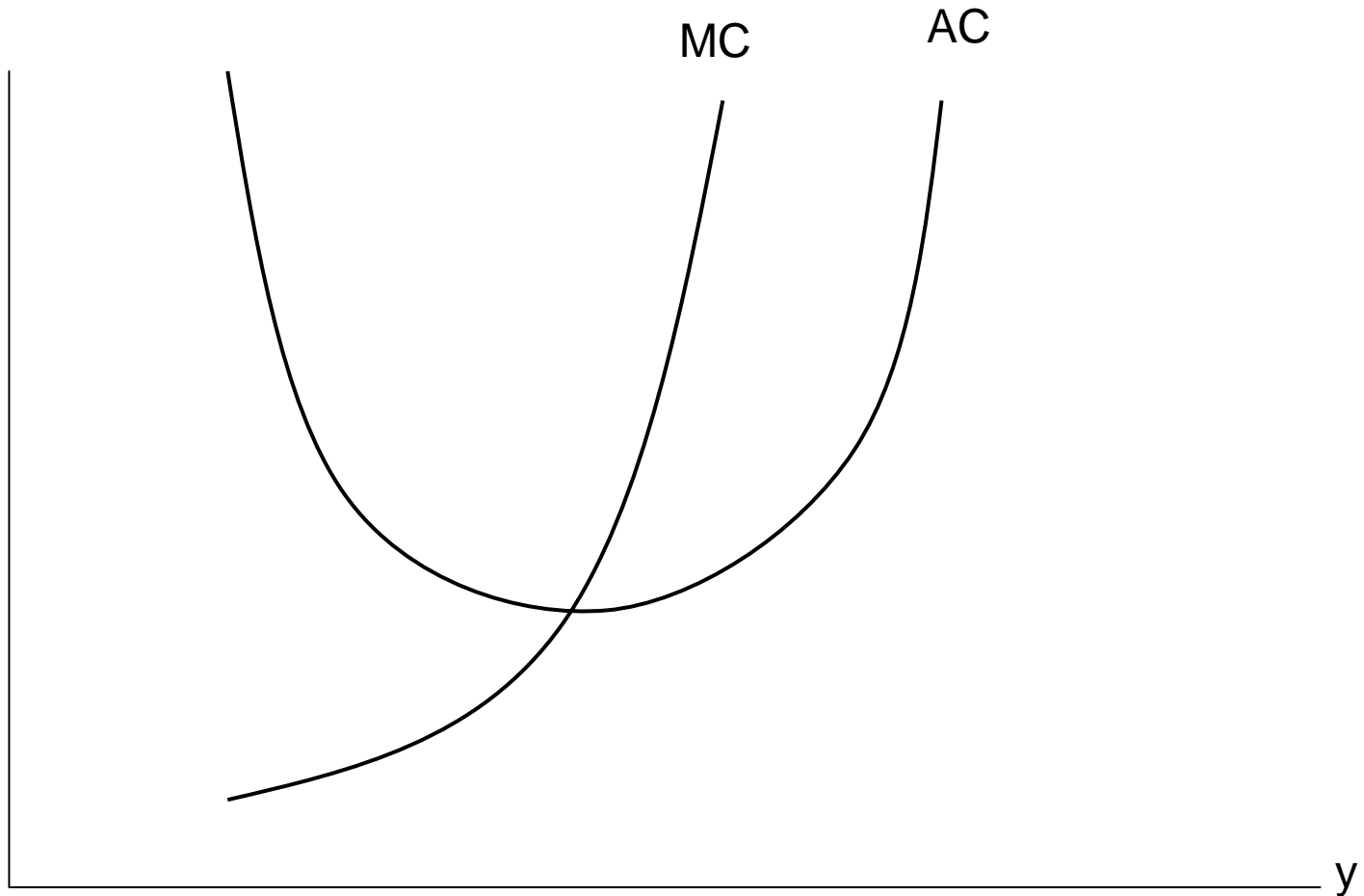
The long - run cost curves is located under the short - run cost cuve.

The former is tangent to the latter.

LAC is the lower envelope of SAC



MC goes through at the point with the lowest AC



MC goes through at the point with the lowest AC

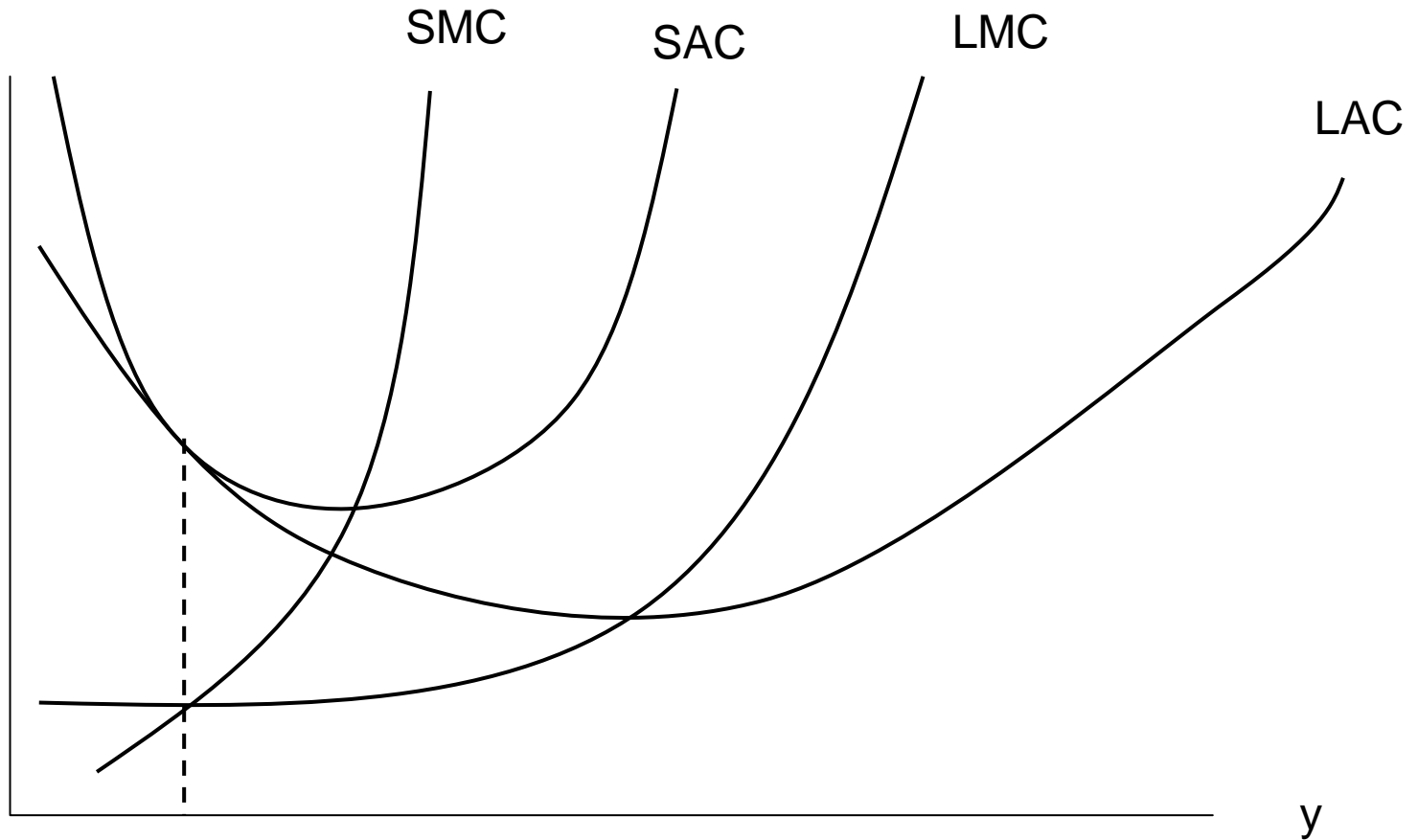
At the lowest point of the AC curve, we have

$$\frac{d}{dy} \left(\frac{c(y)}{y} \right) = \frac{c'(y)y - c(y)}{y^2} = \frac{c'(y)}{y} - \frac{c(y)}{y^2} = 0$$

which is equivalent to

$$c'(y) = \frac{c(y)}{y}$$

$SMC=LMC$ at the tangent point of SAC to LAC



SMC=LMC at SAC=LAC

By definition,

$$c(y) = c_s(y, k(y))$$

Differentiating with respect to y gives

$$\frac{dc(y)}{dy} = \frac{\partial c_s(y, k)}{\partial y} + \frac{\partial c_s(y, k)}{\partial k} \frac{\partial k(y)}{\partial y}$$

Note that $\frac{\partial c_s(y, k)}{\partial k} = 0$, since k is optimally chosen by the firm.

Therefore we are left with,

$$\frac{dc(y)}{dy} = \frac{\partial c_s(y, k)}{\partial y}$$

Firm and Industry Supply

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The Firm Supply Decision

The firm determines its supply to maximize its profit :

$$\max_y py - c(y)$$

The first - order condition is

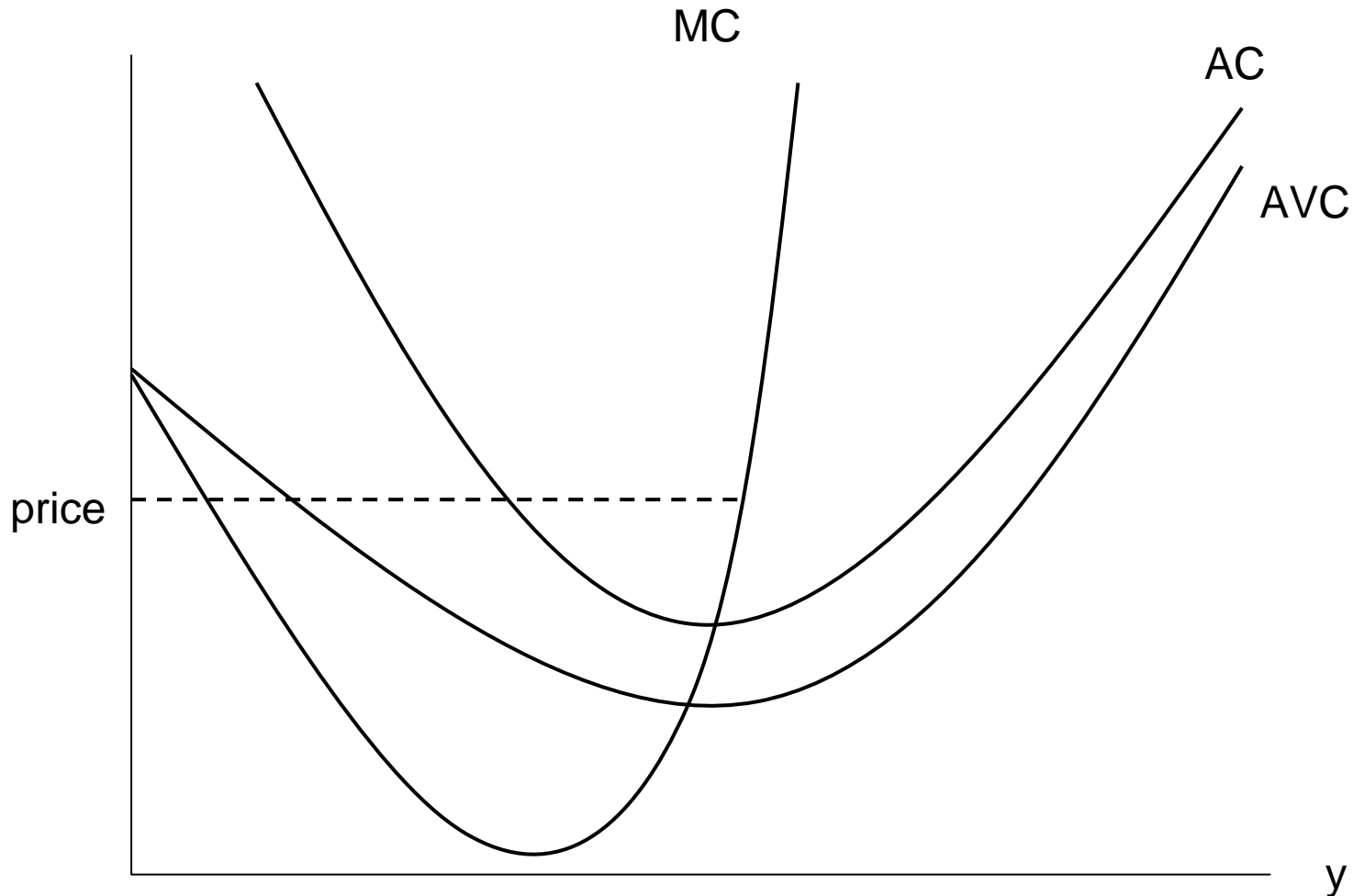
$$p = c'(y).$$

That is, the price is equal to the marginal cost.

The second - order condition is

$$c''(y) > 0.$$

The supply must be on the upward sloping part of the marginal cost curve



Price must be higher than the AVC

The firm chooses to stay in business if

$$-F < py - c_v(y) - F$$

which is equivalent to

$$p > \frac{c_v(y)}{y}$$

That is, the firm stays in the market if the price is higher than the AVC.

Profits and Producer's Surplus

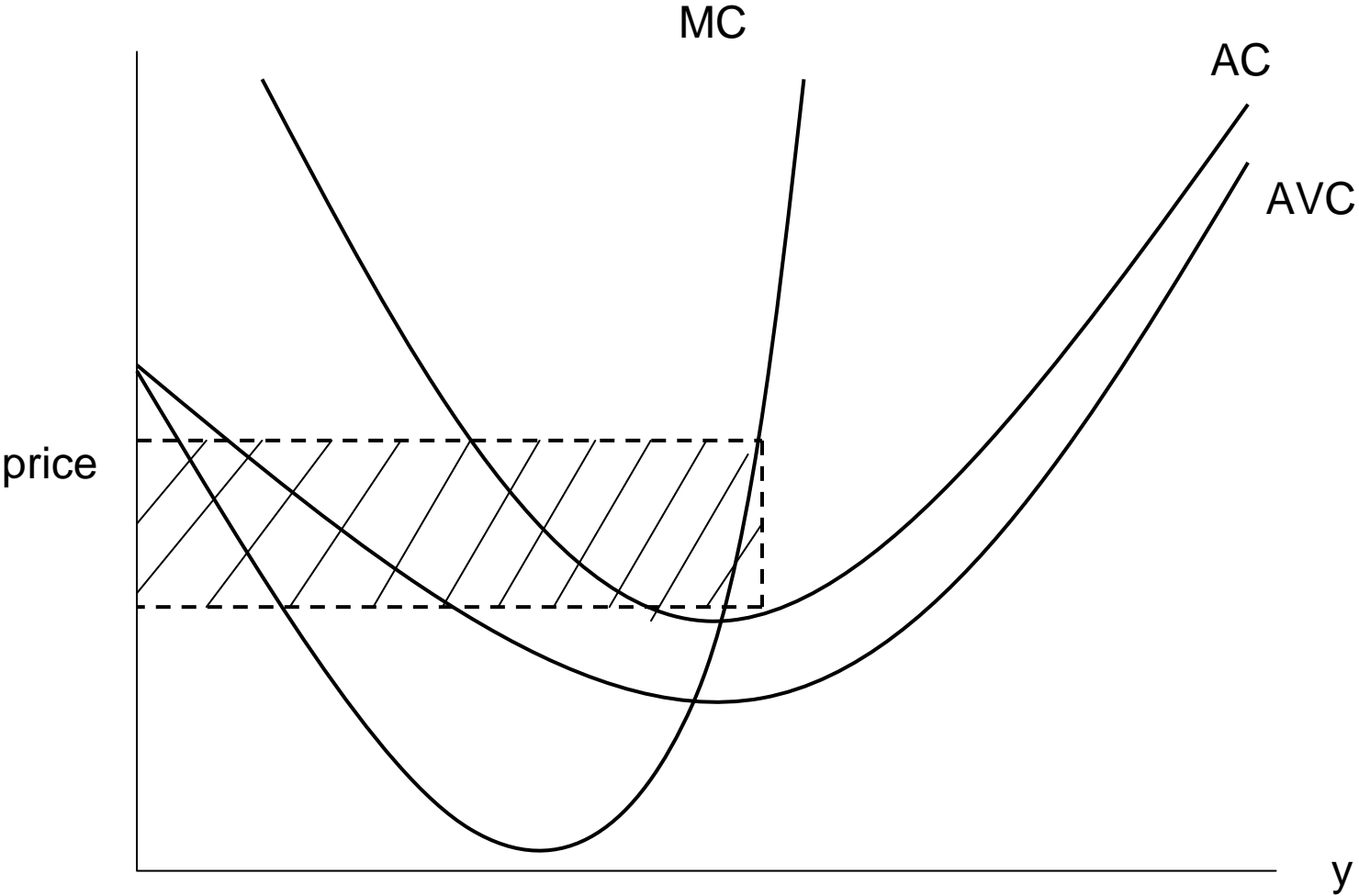
Profits :

$$\pi = py - c(y)$$

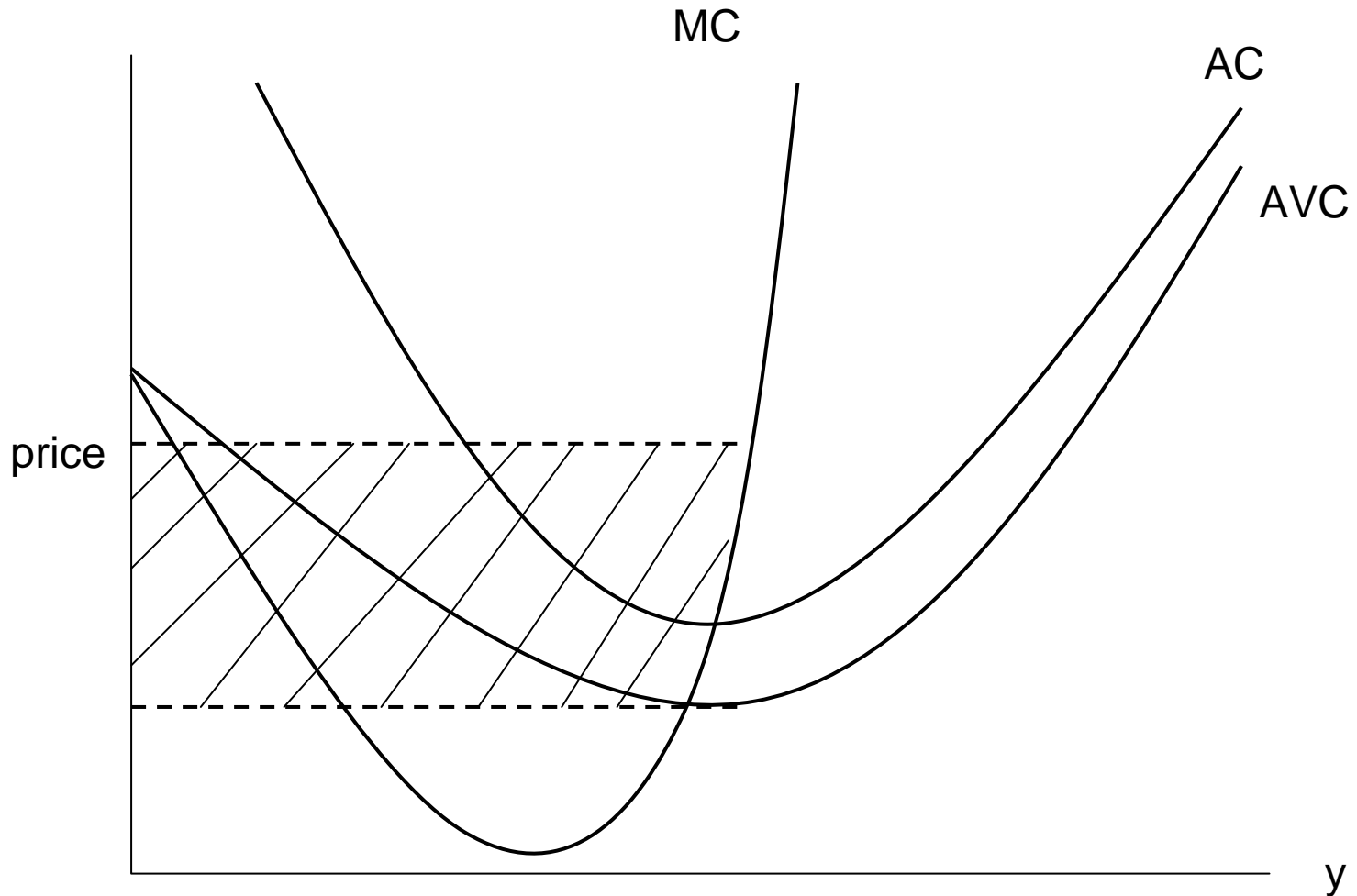
Producer's Surplus :

$$PS = \int_0^y [p - c'(x)] dx$$

Profits



Producer's Surplus



The Industry Supply

- The Industry Supply is the horizontal sum of each firm supply.