

Endogenous Repo Cycles

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Motivation

- repurchase agreement (repo) is a form of loans where
 - a borrower sells a security to a lender for cash
 - the borrower buys back the security at a promised date
- the 2007-2008 crises started with runs in the repo market
 - Gorton and Metrick (2012, 2014)
 - Gorton and Ordonez (2014)

Stylized Facts

- runs in the repo market were associated with a drastic increase in the repo haircut (Gorton and Metrick, 2012)
 - haircut is the difference between an asset value and a loan
- financial crises are typically preceded by credit booms (Gorton and Ordóñez, 2014)
 - a theory of crises on repo should also explain these facts

Our Paper

- we construct a simple random matching model which
 - has **endogenous emergence** of collateralized-loans
 - has **endogenous fluctuations** in the probability of trade, the value of collateral, and its haircut (runs)

Literature

- theories of collateral
 - Kiyotaki and Moore (1997), Ferraris and Watanabe (2011)
- new monetarist approach to monetary economics
 - Kiyotaki and Wright (1989), Kocherlakota (1998)

The Environment

- discrete, infinite time horizon $t = 1, 2, \dots$
- two types of agents, unit measure of **borrowers** and **lenders**
 - each lender has a unit of **land** (durable, divisible)
- borrowers have a better production technology
 - a lender can costlessly produce 1 unit of perishable **fruit** from a unit of land
 - a borrower can produce $a > 1$ units

- each borrower has a unit of collateral good
 - collateral is durable, indivisible
 - cannot be produced nor traded (we can relax this)
 - yields utility $y > 0$ in terms of fruit at end of each period
- agents
 - have a common discount factor $\delta \in (0, 1)$
 - have a linear utility from fruit
 - cannot commit to future actions

Random Pairwise Matching

- at start of each period, each borrower finds a lender with probability 1 at random
 - we can relax this assumption
- in each match,
 - the lender decides how much **land to lend**, $q \in [0, 1]$ in exchange for the borrower's collateral
 - the borrower decides how much **fruit to give**, $r \in [0, aq]$, in order to take back the collateral

- if collateral goes back to the borrower's hand, the payoff to
 - the lender is $(1 - q) + r$ and the borrower is $aq - r + y$
 - period welfare is $1 + (a - 1)q + y$
- at end of the period, all borrowers and lenders separate

- we focus on borrowers' incentives to default, i.e., the borrower can choose $r = 0$ and give up collateral
 - we assume that the lender cannot escape with collateral (we also consider lenders' default in our previous work)

History and Equilibrium

- the history of actions is *private*
 - lenders do not know what r borrowers have chosen in past
 - lenders know borrowers' collateral holdings
- as is standard, the equilibrium notion is *perfect Bayesian equilibrium* (simply, equilibrium henceforth)
- each agent maximizes the discounted lifetime utility (normalized by $1 - \delta$)

No Trade Equilibrium

- no trade is always an equilibrium outcome
 - given $q = 0$, the borrower is indifferent between handing collateral and not
 - given collateral not handed, the lender chooses $q = 0$, otherwise the borrower would choose $r = 0$ (such a deviation will not be detected because of private history)

Repo Strategies

- an endogenous fraction $m_t^* \in [0, 1]$ of the pairs trade using collateral
 - sunspot m_t is drawn for each pair from i.i.d. uniform on $[0, 1]$ prior to trade. trade if and only if $m_t \leq m_t^*$
- lenders choose the highest possible lending volume that satisfies borrowers' incentives (not to default), denoted as q_t^*
 - no trade if a borrower does not have collateral
- borrowers choose the smallest possible return that satisfies lenders' incentives (to trade), denoted as r_t^*

Stationary Equilibria

- we first look for stationary equilibria in which (m_t^*, q_t^*, r_t^*) does not depend on t , denoted as (m^*, q^*, r^*)

Proposition 1. *Let $1 - \delta a > 0$. Then,*

- *if $y \in [1 - \delta a, \infty)$, then the first best, $(q^*, m^*) = (1, 1)$, is an equilibrium outcome*
- *if $y \in (0, 1 - \delta a)$, then some interior but non-trivial trade*

$$q^* = \frac{y}{1 - \delta - \delta m^*(a - 1)} \in (0, 1)$$

is an equilibrium outcome

Proof of Proposition 1

- **borrowers** will not default if returning r^* units of fruits is preferred to returning no fruit

$$(1 - \delta)(aq^* - r^* + y) + \delta V \geq (1 - \delta)aq^* + \delta \cdot 0$$

where V is the value to a borrower from on-equilibrium path

$$V = m^*(aq^* - r^*) + y$$

- **lenders** will trade if lending is preferred to own production

$$(1 - q^*) + r^* \geq 1$$

□

The Value of Collateral

- the (pre-dividend) value of collateral

$$(1 - \delta)y + \delta V = y + \delta m^* q^* (a - 1)$$

- its intrinsic value + value from future trades using it
 - if a borrower defaults, she would also lose future trades
 - if agents expect low value from future trades, the value of collateral drops today (source of cycles)

Equilibrium Trading Volume

- q^* is increasing in m^*

$$q^* = \frac{y}{1 - \delta - \delta m^*(a - 1)}$$

- increased m^* implies higher value from future trades
- it reduces borrowers' incentives to default
- higher lending volume today is credible

Non-Stationary Equilibria: 2-period cycles

- we look for a 2-period cycle in which
 - odd periods, $m_t^* = m_B^*$ for $t = 1 \pmod 2$
 - even periods, $m_t^* = m_R^*$ for $t = 2 \pmod 2$
 - booms in odd periods, in particular,

$$1 = m_B^* > m_R^*$$

Proposition 2. *Let $1 - \delta^2 a > 0$. If*

$$y \in \left(\frac{1 - \delta^2 a}{1 + \delta a}, \frac{1 - \delta^2 a}{1 + \delta} \right)$$

then a 2-period cycle is an equilibrium outcome with

$$1 = m_B^* > m_R^*$$

and

$$q_B^* < q_R^* = 1$$

The period welfare is higher in booms than in recessions, i.e.,

$$m_B^* q_B^* > m_R^* q_R^*$$

Intuition for $q_B^* < q_R^*$

- borrowers' incentive constraint is

$$(1 - \delta)(aq_t^* - r_t^* + y) + \delta V_{t+1} \geq (1 - \delta)aq_t^*$$

- lenders' incentive constraint is

$$r_t^* + 1 - q_t^* \geq 1$$

- hence, we have

$$\underbrace{(1 - \delta)y + \delta V_{t+1}}_{\text{value of collateral}} \geq \underbrace{(1 - \delta)q_t^*}_{\text{credible lending}}$$

- in booms, V_{t+1} is low because tomorrow is a recession
 - the value of collateral is low
 - highest possible lending is low

□

Haircut

- **haircut** is the difference between the value of collateral and the amount of lending, as a percentage of the collateral value,

$$h_t = \frac{[(1 - \delta)y + \delta V_{t+1}] - (1 - \delta)q_t^*}{[(1 - \delta)y + \delta V_{t+1}]}$$

Proposition 3. *Consider the 2-period cycle equilibrium above, i.e. $1 = m_B^* > m_R^*$ and $q_B^* < q_R^* = 1$ is its outcome. Then, we have*

$$h_R > h_B$$

Proof of Proposition 3

- borrowers' and lenders' incentive constraints imply

$$\underbrace{(1 - \delta)y + \delta V_{t+1}}_{\text{value of collateral}} - \underbrace{(1 - \delta)q_t^*}_{\text{credible lending}} \geq 0$$

- recall haircut is

$$h_t = \frac{[(1 - \delta)y + \delta V_{t+1}] - (1 - \delta)q_t^*}{[(1 - \delta)y + \delta V_{t+1}]} = \frac{\text{borrowing constraint}}{\text{value of collateral}}$$

- the borrowing constraint is tighter in booms (binding)

$$(1 - \delta)y + \delta V_R = (1 - \delta)q_B^*$$

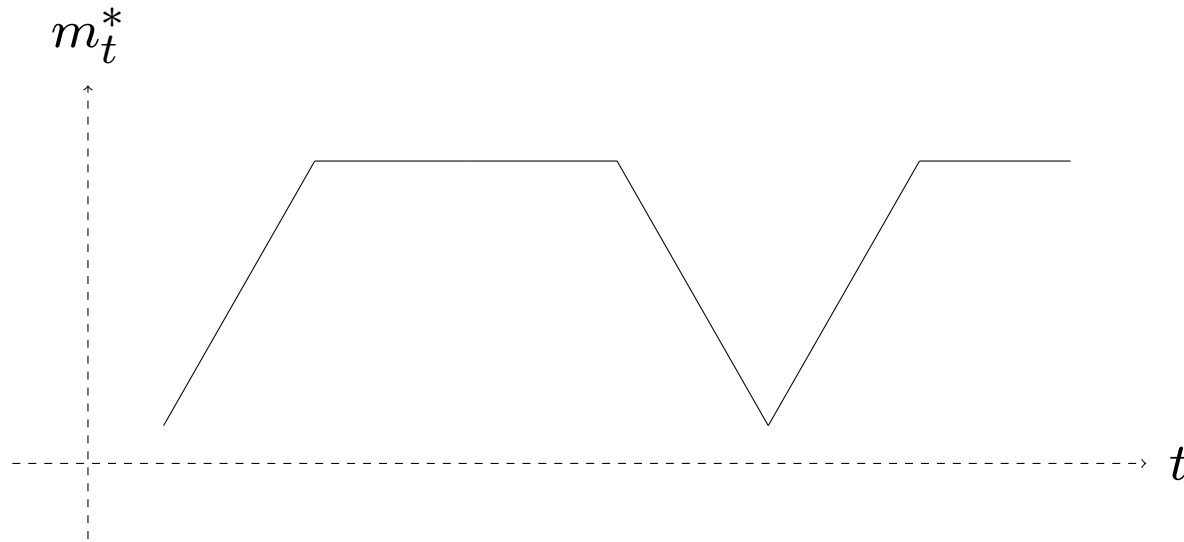
- hence, we have

$$h_R > h_B = 0$$

□

A Brief Summary of 3-period Cycles

- consider $1 = m_3^* = m_2^* > m_1^* > 0$ (period 1 is a recession)



Proposition 4. *Let $1 > \delta^3 a$. If*

$$y \in \left[\frac{1 - \delta^3 a^2}{1 + \delta(1 + \delta)a}, \frac{1 + \delta^2 - \delta^2(1 + \delta)a}{1 + \delta(1 + \delta)} \right]$$

then a 3-period cycle is an equilibrium outcome with

$$1 = m_3^* = m_2^* > m_1^* > 0$$

and

$$q_3^* < 1 = q_2^* = q_1^*$$

- trading volume collapses right before a recession, i.e., $q_3^* < 1$
 - borrowers' incentive constraint is tightest in period 3
(“tomorrow will be a recession”)

$$\underbrace{(1 - \delta)y + \delta V_1}_{\text{low collateral value}} = (1 - \delta)q_3^*$$

- second tightest in period 2
(“day after tomorrow will be a recession”)

Proposition 5. *Consider the 3-period cycle above. Then, generically, haircut is highest in recessions, is second highest right after recessions, and is lowest right before recessions, i.e.,*

$$h_1 > h_2 > h_3$$

Conclusion

- in belief-dependent equilibria, even though there is no time variation in fundamentals, the repo market endogenously fluctuates over time
- if borrowers expect higher value from future trades, they have less incentives to default today, resulting in higher lending today
- the repo haircut is higher in a recession, because the value of future trades, and hence, the value of collateral is higher expecting a future boom