

Money and banking in an overlapping generations economy

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Friedman rule in the OLG model

- Monetary OLG models (CIA/MIUF/Random relocation)
 - Authors including Abel (1987), Freeman (1987), Yakita (1989), Crettez et al. (2002) study the monetary policy in the OLG models.
 - Bhattacharya et al. (JME 2005): a deviation from the **Friedman rule** of setting the nominal interest rate (i) to zero can improve welfare because it generates intergenerational wealth transfers.
- Lagos and Wright (JPE 2005, henceforth LW): one of the advantages of the search models over the existing models is that the role of money is explicit in the search models (microfoundation).
- We study the monetary OLG model with search, and focus on the roles of **financial intermediation** .

Money and banking in the search model

- Berentsen, Camera and Waller (JET 2007, henceforth BCW) incorporate financial intermediation into LW.
 - Day sub-period: Decentralized markets (**DM**) open. Before trading, the individual can borrow money or make deposit at the bank.
 - Night sub-period: Centralized market (**CM**) opens. The buyer pays interest to the seller through the bank.
- Results of BCW
 - ① The optimal monetary policy follows the Friedman rule. Under the rule, financial intermediation does not affect the allocation/welfare.
 - ② Quantity in the DM (q) is a decreasing function of i .
 - ③ The banking sector raises welfare when i is positive.

Research motivation

- Check whether the results in the BCW hold in the OLG model with search.
 - 1 Is the Friedman rule still optimal?
 - 2 Is the financial intermediation beneficial?
- The main differences between infinite horizons model and the OLG model:
 - 1 Intergenerational transfer.
 - 2 Quasi-linearity of the utility function in the CM.

Literature on related models

- Zhu (JET 2008) constructs OLG model with search.
 - The utility function in the CM is strictly concave.
 - Find that optimal money growth rate is positive.
 - Constraints on money never bind and then the optimal nominal interest rate is zero.
- Jacquet and Tan (JME 2012) study OLG model with idiosyncratic shock and find that assets become heterogeneous in liquidity.
- We consider financial intermediation in the OLG model with search. The set-up is close to Zhu (2008) and Jacquet and Tan (2012).

Set-up

- Time is discrete. Each period has two sub-periods (day/night). DMs open in the day sub-period, and CM opens at night.
- In each period, a continuum of individual with measure one is born at night, and lives for three sub-periods.
 - 1 1st stage (night): receive $\bar{\omega}^y$ units of general good, consume and hold money. Utility function: $U(C)$.
 - 2 2nd stage (day): become a buyer with prob 0.5, and a seller with prob 0.5. Use the bank, and trade. Utility function: $u(q), -c(q)$.
 - 3 3rd stage (night): receive $\bar{\omega}$ units of general good, pay or receive interest, consume and leave.
- Transactions in the DM need money. Central bank prints money. New money is injected to the young.

Seller's problem in the DM

- A seller with m units of money sells q units of the DM goods to a buyer. The price is p .
- The seller makes a deposit all the money in the bank, and solves

$$V_s(m) = \max[-c(q) + U(C^s)], \quad (1)$$

$$C^s = (1 + i)\phi m + \phi p q + \bar{w}, \quad (2)$$

where ϕ is the price of money.

- The first order condition (FOC)

$$c'(q) = \phi p U'(C^s). \quad (3)$$

Buyer's problem in the DM

- A buyer with m units of money buys q units of the DM goods.
- The buyer borrows $pq - m$ units of money from the bank:

$$V_b(m) = \max[u(q) + U(C^b)], \quad (4)$$

$$C^b = -(1+i)\phi(pq - m) + \bar{\omega}. \quad (5)$$

- The FOC

$$u'(q) = (1+i)\phi p U'(C^b). \quad (6)$$

Banking sector

- Accept nominal deposit at rate i , and make loan at rate i .
- The bank can force repayment at no cost, and is subject to the free entry conditions.
- In equilibrium, $pq - m = m$. Thus

$$pq = 2m. \quad (7)$$

- In models without banking, $pq = m$.
- We focus on the case with positive i .

Young individual in the CM

- The individual solves ($\delta < 1$:discount factor)

$$V = \max U(C^y) + \frac{\delta}{2}[V_b(m_{+1}) + V_s(m_{+1})], \quad (8)$$

$$\text{s.t. } \bar{\omega}^y = \phi m_{+1} + C^y - \tau \phi M_{+1}. \quad (9)$$

- Growth rate of money supply is τ : $M_{+1} = (1 + \tau)M$.
- The FOCs on money (**Euler equation**)

$$U'(C^y) = \delta(1 + i) \frac{\phi_{+1}}{\phi} \left\{ \frac{U'(C_{+1}^b) + U'(C_{+1}^s)}{2} \right\}. \quad (10)$$

Competitive equilibrium

- Real balances:

$$\rho = \phi m. \quad (11)$$

- Equilibrium allocation: $(i, p, \rho, q, C^y, C^b, C^s)$
- The budget constraints:

$$\text{1st} \quad : \quad \bar{\omega}^y = C^y + \rho, \quad (12)$$

$$\text{3rd (b)} \quad : \quad C_{+1}^b = -(1+i)(\phi_{+1}p_{+1}q_{+1} - \rho_{+1}) + \bar{\omega}, \quad (13)$$

$$\text{3rd (s)} \quad : \quad C_{+1}^s = (1+i)\rho_{+1} + \phi_{+1}p_{+1}q_{+1} + \bar{\omega}. \quad (14)$$

- The expenditure on the DM good is twice as much as the money balances:

$$\phi pq = 2\rho. \quad (15)$$

The FOCs

- The FOCs in the first stage

$$U'(C^y) = 0.5 \frac{1+i}{1+\tau} \frac{\rho_{+1}}{\rho} \delta \{U'(C_{+1}^b) + U'(C_{+1}^s)\}. \quad (16)$$

- The FOCs in the second stage

$$q_{+1}u'(q_{+1}) = 2\rho_{+1}(1+i)U'(C_{+1}^b), \quad (17)$$

$$q_{+1}c'(q_{+1}) = 2\rho_{+1}U'(C_{+1}^s). \quad (18)$$

Steady state

- The steady state (i, ρ, q, C^b, C^s) solve

$$U'(\bar{\omega}^y - \rho) = \delta \frac{1+i}{1+\tau} \frac{U'(C^b) + U'(C^s)}{2}, \quad (19)$$

$$qc'(q) = 2\rho U'(C^s), \quad (20)$$

$$qu'(q) = 2\rho(1+i)U'(C^b), \quad (21)$$

$$C^b = -(1+i)\rho + \bar{\omega}, \quad (22)$$

$$C^s = (3+i)\rho + \bar{\omega}. \quad (23)$$

- Welfare $(s(q) = u(q) - c(q))$: surplus

$$W = U(\bar{\omega}^y - \rho) + \delta \frac{s(q) + U(C^b) + U(C^s)}{2}. \quad (24)$$

Steady state welfare

- Suppose that $U(x) = x$. FOCs in the DM

$$\frac{u'(q)}{c'(q)} = 1 + i = \frac{1 + \tau}{\delta} \quad (25)$$

$$qc'(q) = 2\rho. \quad (26)$$

- The central bank chooses $i \geq 0$ to maximize welfare.

$$W = \bar{\omega}^y - \rho + \delta\{0.5s(q) + \bar{\omega} + \rho\} \quad (27)$$

$$= \bar{\omega}^y + \delta\bar{\omega} + 0.5\delta s(q) - (1 - \delta)\rho. \quad (28)$$

- Effect of nominal interest rate on welfare.

- 1 Reduce the surplus $s(q)$ ($W \downarrow$).
- 2 Reduce the intergenerational transfer ρ ($W \uparrow$).

Optimal monetary policy

Theorem

The Friedman rule is not optimal.

Proof.

Welfare is a function of q :

$$\begin{aligned} W(q) &= \frac{\delta}{2}s(q) - (1 - \delta)\rho(q) + \bar{\omega}^y + \delta\bar{\omega}, \\ \rho(q) &= \frac{1}{2}qc'(q). \end{aligned} \tag{29}$$

The surplus s is maximized if $i = 0$. Since $dq/di < 0$, $dW/di > 0$. □

Equilibrium without bank

- The FOC on money:

$$\phi = \delta\phi_{+1} \frac{1}{2} \left(\frac{u'(\hat{q})}{c'(\hat{q})} + 1 \right). \quad (30)$$

- Opportunity cost of holding money is higher in the model without banking.

$$2i = \frac{u'(\hat{q})}{c'(\hat{q})} - 1. \quad (31)$$

- Real balances are **doubled** in the model without banking:

$$\rho = \hat{q}c'(\hat{q}). \quad (32)$$

Optimal monetary policy

Theorem

Financial intermediation improves welfare (even) under the Friedman rule.

Proof.

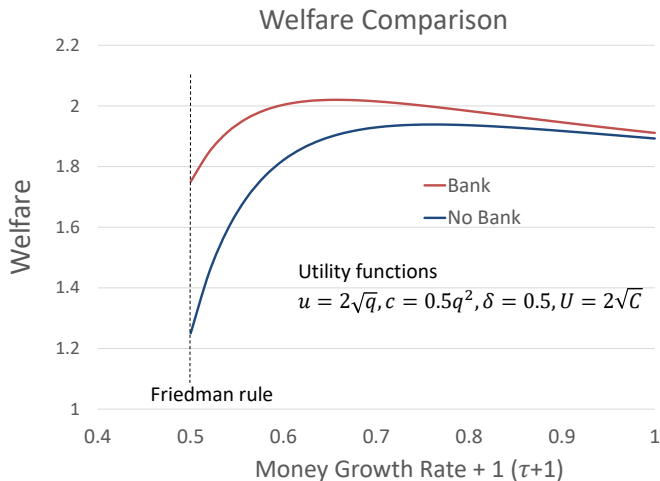
Welfare:

$$W_{bank} = \frac{1}{2}\delta s(q) - (1 - \delta)\frac{qc'(q)}{2} + const \quad (33)$$

$$W_{nobank} = \frac{1}{2}\delta s(\hat{q}) - (1 - \delta)\hat{q}c'(\hat{q}) + const \quad (34)$$

Under the Friedman rule, quantity is efficient. Thus $W_{bank} > W_{nobank}$ under the Friedman rule. □

Figure on welfare



Log utility function

Theorem

Assume that $u(q) = \ln q$, $c(q) = \frac{q^{1+\eta}}{1+\eta}$ and $U(C) = \ln C$. Then the allocation satisfies

$$\rho = \frac{\bar{\omega}}{3(1+i)}, \quad q = \left(\frac{1}{3+2i} \right)^{1/(1+\eta)} \quad (35)$$

If $\bar{\omega}^y / \bar{\omega}$ is less than $1/3$, then the Friedman rule is not optimal.

Theorem

If the government can change the endowment by the intergenerational transfer policy such that $\omega^y + \omega = \bar{\omega} + \bar{\omega}^y$, the rule is still nonoptimal if η is sufficiently high.

Conclusion

- We construct the search theoretic OLG model with banking, and focus on the monetary policy maximizing the steady state welfare.
 - We find that the Friedman rule may not be optimal.
 - Non-optimality of the Friedman rule holds even if the intergenerational transfer policy is available.
- Results differ from Berentsen, Camera and Waller.