

## Asset Equilibria with Indivisible Goods

Han Han

Peking University

Benoît Julien

UNSW Australia

Asgerdur Petursdottir

University of Bath

Liang Wang

UH Manoa

March 9-10, 2018

Search and Matching Kyushu University Japan

# Introduction

- We study equilibria with real assets being the medium of exchange in a New Monetarist framework with indivisible goods.
  - ▶ Many goods are indivisible, especially at the micro level, such as housing, service, bulky items, etc., and buyers may or may not want more than one unit. Here focus on buyers demanding only one unit.
- With indivisible goods one loses an intensive margin of adjustment: consequences in equilibrium?
- Assets have positive or negative dividend: implications in equilibria, compared to money?
- Study two different pricing mechanisms: generalized Nash bargaining and competitive search.

## Findings and Literature

- **Divisible goods, divisible assets:** Lagos and Rocheteau (2009), Rocheteau and Wright (2013)
  - ▶ It matters for equilibrium which trading mechanism is used to determine price and whether the asset dividend is positive or negative.
- **Indivisible goods, divisible money:** Green and Zhou (1998), Jean et al. (2010), Han et al. (2016)
  - ▶ Price posting with random search - multiplicity of monetary equilibria.
  - ▶ Assets vs. money and credit - endogenous vs. exogenous/zero cost of holding liquidity.
- **Divisible goods, indivisible money:** Bargaining: Shi (1995), Trejos and Wright (1995); Competitive Search: Julien, Kennes and King (2008)
  - ▶ It matters for equilibrium where the indivisibility lies, goods side or money side.
- **Effect of inflation:** Lagos and Rocheteau (2005), Liu et al. (2011)
  - ▶ Search intensity - effect on the extensive margin, “hot potato” effect

# Environment

- Time is discrete and continues forever.
- A continuum of buyers with measure  $N$  and sellers with measure 1 live forever.
  - ▶ implying a buyer-seller ratio of  $N$ .
- Each period is divided into two subperiods:
  - 1 Decentralized market (DM): trade of the indivisible goods.
  - 2 Centralized market (CM): agents discount between periods with discount factor  $\beta = \frac{1}{1+r} \in (0, 1)$ .

## Environment

Agents' preferences within a period:

$$\text{Buyers} : U(x_t) - h_t + u$$

$$\text{Sellers} : U(x_t) - h_t - c$$

- $x_t$ : CM consumption of divisible general goods;
- $h_t$ : CM labor ( $x_t$  is produced one-to-one from  $h_t$ );
- $u$ : DM utility from consuming the indivisible goods;
- $c$ : Cost of producing the DM indivisible goods.

In the DM  $\rightarrow$  meetings occur according to a general matching technology.

- Matching rate for sellers:  $\alpha(n)$ ; for buyers:  $\alpha(n)/n$ ; given a buyer-seller ratio  $n \leq N$  ( $n$  being the number of active buyers).
- Standard assumptions on  $\alpha(n)$ .

# Assets

- Assume agents in the DM cannot commit, anonymous and no record keeping or enforcement mechanisms.
  - ▶  $\Rightarrow$  Buyers need to bring a medium of exchange into the DM to trade with a seller.
  - ▶ They use a divisible real asset with fixed supply  $A^s$ , which pays dividend  $\rho$ , as the medium of exchange.
- The cost of holding assets,  $s = \varphi_t / \beta(\varphi_{t+1} + \rho) - 1$ , depends on the number of active buyers when the asset is priced above its fundamental value.
  - ▶ Asset vs. money: the cost of holding money,  $i = \phi_t / \beta\phi_{t+1} - 1$ , is given exogenously; the cost of using credit, if available, is zero.

# Assets

- Consequences for equilibria:
  - ▶ The asset has an intrinsic value - therefore, if priced fundamentally, sellers will want to hold the asset.
  - ▶ Competitive search: the price of the indivisible good is affected by the value of the dividend.
  - ▶ Not with bargaining: coordination failure and multiple equilibria when  $\rho < 0$ .

# Centralized Market

- Buyers that bring assets  $a$  to the CM solve

$$W_t^b(a) = \max_{x, h, \hat{a}} \left\{ U(x) - h + \beta V_{t+1}^b(\hat{a}) \right\}$$

s.t.  $x = (\varphi_t + \rho)a + h - \varphi_t \hat{a}$

$$W_t^b(a) = U(x^*) - x^* + (\varphi_t + \rho)a + \max_{\hat{a}} \left\{ \beta V_{t+1}^b(\hat{a}) - \varphi_t \hat{a} \right\}$$

where  $\varphi_t$  is the CM price of real assets in terms of  $x_t$ .

- Sellers don't need assets to trade but they hold assets iff fundamentally priced

$$W_t^s(a) = U(x^*) - x^* + (\varphi_t + \rho)a + \max_{\hat{a}} \left\{ \beta V_{t+1}^s(\hat{a}) - \varphi_t \hat{a} \right\}.$$



## Decentralized Market

- Buyer's value function in the DM

$$V_t^b(a) = \frac{\alpha(n)}{n} \left[ u + W_t^b \left( a - \frac{p}{\varphi_t + \rho} \right) \right] + \left[ 1 - \frac{\alpha(n)}{n} \right] W_t^b(a),$$

where  $p$  is the CM price of the indivisible DM good.

- Simplifying:

$$\text{Buyers : } V_t^b(a) = \frac{\alpha(n)}{n} (u - p) + W_t^b(a)$$

$$\text{Sellers : } V_t^s(a) = \alpha(n) (p - c) + W_t^s(a)$$

# Bargaining

- The generalized Nash bargaining problem

$$\begin{aligned} & \max_p (u - p)^\eta (p - c)^{(1-\eta)} \\ \text{s.t. } & p \leq (\varphi_t + \rho)a, u - p \geq 0, p - c \geq 0 \end{aligned}$$

- When  $s > 0$ , the feasibility constraint  $p \leq (\varphi_t + \rho)a$  is binding and  $c \leq p = (\varphi_t + \rho)a \leq \bar{p}^B$ .
  - ▶  $\bar{p}^B = (1 - \eta)u + \eta c$  is the unconstrained bargaining solution.
- Any negotiated price  $p \in [c, \bar{p}^B]$  is a potential bargaining solution.

## Bargaining

- Substituting  $V_{t+1}^b$  into  $W_t^b$  and the buyer's problem is simplified to:

$$\tilde{V}^b(\hat{a}; n, s_t) \equiv \max_{\hat{a} \in [\underline{a}, \bar{a}]} \beta \left\{ \frac{\alpha(n)}{n} [u - (\varphi_{t+1} + \rho) \hat{a}] - s_t (\varphi_{t+1} + \rho) \hat{a} \right\}$$

where  $\underline{a} = c / (\varphi_{t+1} + \rho)$  and  $\bar{a} = [(1 - \eta)u + \eta c] / (\varphi_{t+1} + \rho)$ .

- Buyer's surplus decreases in  $\hat{a}$ , and the buyer has a 'first-mover' advantage

⇒ the optimal asset holding decision reduces to:

$$p = \hat{a}^*(\varphi_{t+1} + \rho) = c.$$

- ▶ Buyer receives surplus of  $u - c$  instead of  $\eta(u - c)$ .
- ▶ Buyer commits to not paying more than the seller's cost. The solution is akin to a take-it-or-leave-it offer to sellers.

## Entry and Equilibrium

- The measure of DM buyers  $n^*$  is determined by the free entry condition  $\tilde{V}^b(n, s_t) = 0$  with

$$\tilde{V}^b(n, s_t) = \beta \left[ \frac{\alpha(n)}{n} (u - c) - s_t c \right]$$

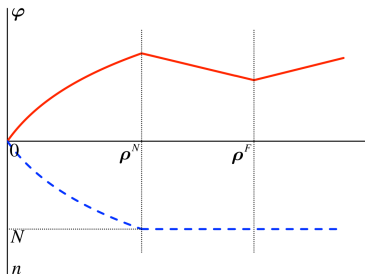
- We solve for stationary equilibrium using

$$L^s(s) = [\varphi(s) + \rho] A^s = n(s)c = L^d(s)$$

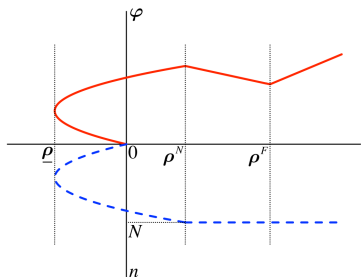
$n(s)$  determined by free entry condition, then back out  $\varphi$  and  $n$ .

- Assets are priced fundamentally if  $\varphi_t = \beta(\varphi_{t+1} + \rho) \Rightarrow \varphi^F = \rho/r$ ,  $s = 0$ .
- When assets are scarce:  $\varphi = cn/A^s - \rho > \varphi^F$ .

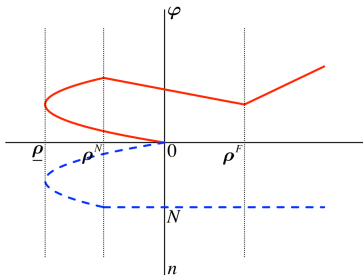
# Equilibrium with Bargaining



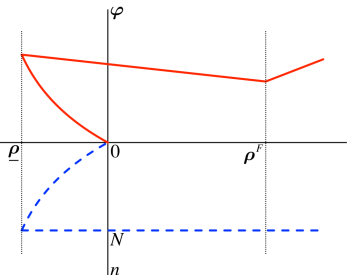
a.  $\underline{\rho} = 0 \leq \rho^N$



b.  $\underline{\rho} < 0 < \rho^N$



c.  $\underline{\rho} < \rho^N \leq 0$



d.  $\underline{\rho} = \rho^N < 0$

## Bargaining: Comparative Statics

**Proposition:** In the stable SE with bargaining: (i) for  $\mathcal{A} \geq \rho^F A^s$ , we have  $\partial\varphi/\partial\mathcal{A} > 0$  and  $\partial n/\partial\mathcal{A} = 0$ ; (ii) for  $\mathcal{A} \in [\rho^N A^s, \rho^F A^s)$ ,  $\partial\varphi/\partial\mathcal{A} < 0$  and  $\partial n/\partial\mathcal{A} = 0$ ; (iii) for  $\mathcal{A} \in (\underline{\rho} A^s, \rho^N A^s)$ ,  $\partial\varphi/\partial\mathcal{A} > 0$  and  $\partial n/\partial\mathcal{A} > 0$ . In the unstable SE with bargaining, for  $\mathcal{A} \in (\underline{\rho} A^s, 0)$ ,  $\partial\varphi/\partial\mathcal{A} < 0$  and  $\partial n/\partial\mathcal{A} < 0$ .

- Extensive margin starts to adjust in case (iii), when  $\rho < 0$ , “hot potato” effect vs. “poison apple” effect.
  - ▶ Liu et al. (2011) studies “hot potato” effect through the extensive margin in a monetary model with bargaining.

## Bargaining: Credit

- Buyers in the CM

$$W_t^b(d) = \max_{x,h} \left\{ U(x) - h + \beta V_{t+1}^b \right\} \text{ s.t. } x = h - d.$$

$d$  is DM debt:  $d = p$  DM goods price if trade,  $d = 0$  otherwise.

- Buyers in the DM

$$V_t^b = \frac{\alpha(n)}{n} \left[ u + W_t^b(p) \right] + \left( 1 - \frac{\alpha(n)}{n} \right) W_t^b(0).$$

- The bargaining solution is  $p = (1 - \eta)u + \eta c$ .
- All buyers are active in the DM since using credit is costless.

## Bargaining: Money

- Buyers in the CM

$$\begin{aligned}W_t^b(m) &= \max_{x, h, \hat{m}} \left\{ U(x) - h + \beta V_{t+1}^b(\hat{m}) \right\} \\ \text{s.t. } x &= \phi_t(m + T) + h - \phi_t \hat{m}\end{aligned}$$

$T$  is net transfers from government.

- Buyers in the DM

$$V_t^b(m) = \frac{\alpha(n)}{n} \left[ u + W_t^b(m - p) \right] + \left( 1 - \frac{\alpha(n)}{n} \right) W_t^b(m).$$

- The bargaining solution is  $c \leq p = \phi m \leq \bar{p}^B = (1 - \eta)u + \eta c$  as in the case with credit.



## Bargaining: Money

- Depending on  $i$ , some buyers are active in the DM, and everyone carries  $\phi m = c$ . The solution is akin to take-it-or-leave-it offer to sellers.
- When  $i \leq \bar{i}^N$ , all buyers participate in the DM.
  - ▶ Money is superneutral.
  - ▶ Introducing lotteries does not overturn the superneutrality.
- When  $i^B > i > \bar{i}^N$ ,  $n^* < N$ .
  - ▶ Money is no longer superneutral since inflation reduces the number of buyers  $\Rightarrow$  adjustment through the extensive margin by increasing buyers' probability of trade.

## Bargaining: Asset vs. Credit/Money

- Equilibrium price with credit is always higher than with money or asset.
  - ▶ With money or asset, buyers bring resources into the DM ex ante. However, with credit, buyers do not have the power to commit to paying  $c$  ex ante.
- When  $\rho = 0$ , assets are equivalent to money with a constant supply.
- With money, stationary equilibrium is unique. A higher participation in the DM implies lower probability to trade for buyers. Since the cost of holding money is not adjusting with  $n$ , buyers are strictly worse off, and the above coordination does not exist.
- When  $\rho > 0$ , an equilibrium with assets always exists. However, monetary equilibrium may not exist with deflation, if the surplus from trade is small enough, since the cost of holding money is independent of  $n$ .

# Competitive Search

- Setup follows Wright, Julien, Kircher and Guerrieri (2017)
- A seller chooses a price to post by solving:

$$\begin{aligned}\tilde{V}^s(p, n) &= \max_{p, n} \alpha(n) (p - c) \\ \text{s.t. } \tilde{V}^b(p, n) &= \frac{\alpha(n)}{n} (u - p) - sp \geq \Omega, \\ p &\leq (\varphi_t + \rho) a.\end{aligned}$$

$\Omega$  is the equilibrium expected utility of a buyer in the DM.

# Competitive Search

- We can simplify the seller's problem

$$\max_n \alpha(n) \left[ \frac{\alpha(n)u - n\Omega}{\alpha(n) + ns} - c \right]$$

- The seller's optimal price  $p^c$  satisfies

$$p^c(n^*) = \frac{\alpha(n^*) \{ [1 - \varepsilon(n^*)] u + \varepsilon(n^*) c \} + \varepsilon(n^*) n^* s c}{\alpha(n^*) + \varepsilon(n^*) n^* s}$$

## Entry and Equilibrium

- In equilibrium buyers' participation is consistent with

$$\frac{\alpha(n^*)}{n^*}(u - p^c) - sp^c(n^*) = \Omega \geq 0 \Rightarrow n^*(s)$$

- We solve for stationary equilibrium using

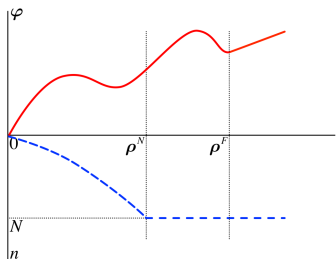
$$L^s(s) = [\varphi(s) + \rho] A^s = n^*(s)p^c(n^*) = L^d(s),$$

with  $n(s)$  determined from free entry

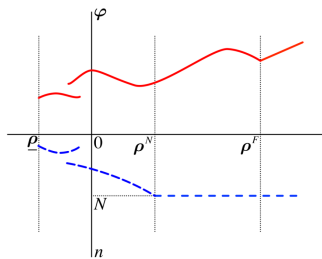
$$\beta \frac{\alpha(n)}{n} [u - p^c(n^*)] = sp^c(n^*).$$

- Then back out  $\varphi(s)$  and  $n(s)$ .

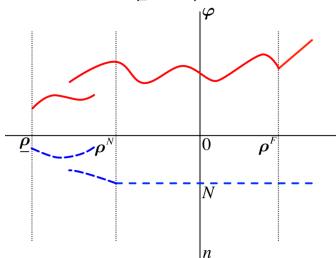
# Equilibrium with Competitive Search



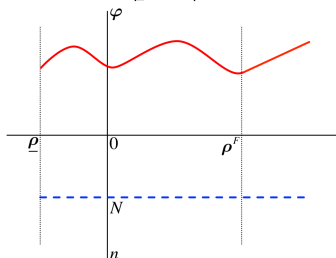
a.  $\underline{\rho} = 0 \leq \rho^N$



b.  $\underline{\rho} < 0 < \rho^N$



c.  $\underline{\rho} < \rho^N \leq 0$



d.  $\underline{\rho} = \rho^N < 0$

## Competitive Search: Comparative Statics

**Proposition:** In the SE with competitive search: (i) for  $\mathcal{A} \geq \rho^F A^s$ , we have  $\partial\varphi/\partial\mathcal{A} > 0$  and  $\partial n/\partial\mathcal{A} = 0$ ; (ii) for  $\mathcal{A} \in [\rho^N A^s, \rho^F A^s)$ ,  $\partial n/\partial\mathcal{A} = 0$  and  $\partial\varphi/\partial\mathcal{A}$  is ambiguous; (iii) for  $\mathcal{A} \in (\underline{\rho} A^s, \rho^N A^s)$ ,  $\partial\varphi/\partial\mathcal{A}$  and  $\partial n/\partial\mathcal{A}$  are ambiguous.

- Extensive margin starts to adjust in case (iii). When  $\rho < 0$  and  $n^* < N$ , the joint effect of “hot potato” and “poison apple” is ambiguous.

## Competitive Search vs. Bargaining

- With price posting, sellers 'move first' and can acquire some of the surplus from trade.
- $p^c$  and buyer's real asset holding depend on  $s/\rho$  and the number of active buyers  $n^*$ .
- For  $\rho > 0$ , there is a unique equilibrium under both competitive search and bargaining, since there does not exist a trade-off between the probability to trade and the cost of holding liquidity.
- For  $\rho < 0$ , multiple equilibria exist with bargaining, but the equilibrium is still unique for  $\rho \in (\rho^N, 0)$  with competitive search.
  - ▶ With bargaining, buyers search randomly and the equilibrium price is the seller's reservation value, independent of the market tightness.
  - ▶ With competitive search, the prices posted by sellers direct the buyers' search behavior and serve as a coordination device.



## Competitive Search: Asset vs. Credit/Money

- If  $s = 0$ , asset is costless to hold, and the DM price reduces to

$$p^c = [1 - \varepsilon(n^*)] u + \varepsilon(n^*) c,$$

the same as the price under credit, and bargaining with credit given  $\eta = \varepsilon(n)$ .

- Lose equilibrium uniqueness with  $\rho < 0$  and  $n^* < N$ .
  - ▶ In monetary models, given  $i$ , competitive search works as a coordination device and eliminate multiple equilibrium.
  - ▶ With assets, the spread  $s$  is similar to the opportunity cost of holding money  $i$ , but endogenously determined by demand and supply of liquidity. Given  $s$ , competitive search still works as a coordination device to eliminate multiplicity. However, there can be equilibria with different values of  $s$  and different levels of entry correspondingly, each generating the same equilibrium payoff for agents, hence multiple equilibria.

# Lotteries

- Let  $\tau \in [0, 1]$  be the probability that the good is produced and traded.
- Under bargaining,  $p^b = uc / (\eta u + (1 - \eta)c) > c$  and  $\tau^b = 1$  in equilibrium.
  - ▶  $p^b$  and  $\tau^b$  do not change w.r.t.  $\rho/s$ .
  - ▶ multiple equilibrium
  - ▶  $\Rightarrow$  lotteries do not lead to equilibrium uniqueness or restore conventional wisdom.
- Under competitive search,  $\tau^c = 1$  and  $p^c$  same in equilibrium.
- With indivisible goods, no adjustment can take place through the intensive margin and the total available surplus from each trade is fixed. With lotteries, agents always want to bargain/post  $\tau = 1$  to maximize the total surplus from trade.

# Conclusions

- The payment medium matters for equilibrium outcomes.
- (Generically) unique equilibrium with credit and money - possible multiplicity with assets if negative dividend.
- The exogenously determined pricing mechanism used matters for equilibrium outcomes.
  - ▶ 'First-mover' advantage with indivisible goods.
- Indivisibility of the DM good matters.
  - ▶ The importance of the results rely on the importance of indivisibility.