

# Middlemen and Resale

Ryoichi Imai

Kyushu University

August 15, 2012

- New Good Market - Centralized with no/less search friction
  - you can immediately buy whatever you want.
  - you can immediately sell whatever you need to sell.
- Resale Market - Decentralized with more search friction
  - you need to spend time to acquire what you want.
  - or to sell what you need to sell.,
- Examples
  - Houses and automobiles, books and CDs
  - internet retailers/auction sites
    - Amazon, e-bay, Yahoo! Auction

- There are considerable amount of works in the IO context, such as Courty (2003) and Moller and Watanabe (2009), among others.
- But no (major or minor) works have been done in the Diamond-Mortensen-Pissarides type search-theoretic framework.
- Imai (2011, mimeo.) presents a tentative work, while inventory constraint is imposed on the sellers, in the sense that each seller is allowed to hold one unit of the used good.
- Watanabe (2006 and others) develops a search model of middlemen, in which middlemen holds multiple inventory.

# The Present Paper

- 1 Two markets: new and resale
- 2 It relaxes the inventory constraint to allow for multiple inventory.
- 3 Middlemen mediate trades in the resale market.
- 4 Main Result: Resale creates a higher gain not only for consumers but for suppliers.

- 1 Environments
- 2 No resale
- 3 Middlemen in the resale market
- 4 Comparative Statics
- 5 Conclusion

# Environment 1

- Time is continuous.
- There are three markets; a new and two resale markets.
- A single good is supplied in the new good market, which is frictionless, and monopolized or competitive.
- The resale market has two divisions: sell and buy markets.
  - ① In the *frictionless* sell-market, the used goods are purchased by the middlemen.
- There are numerous consumers, heterogeneous in the valuation of the new good, while they are identical in the valuation of the used good.
- A random shock lowers good's quality.
  - The 1st shock turns a new good to an old one.
  - The 2nd shock turns an old good to garbage/waste.

# Environment 1

- Time is continuous.
- There are three markets; a new and two resale markets.
- A single good is supplied in the new good market, which is frictionless, and monopolized or competitive.
- The resale market has two divisions: sell and buy markets.
  - ① In the *frictionless* sell-market, the used goods are purchased by the middlemen.
  - ② In the *frictional* buy-market, consumers can buy the used goods from the middlemen.
- There are numerous consumers, heterogeneous in the valuation of the new good, while they are identical in the valuation of the used good.
- A random shock lowers good's quality.
  - The 1st shock turns a new good to an old one.
  - The 2nd shock turns an old good to garbage/waste.

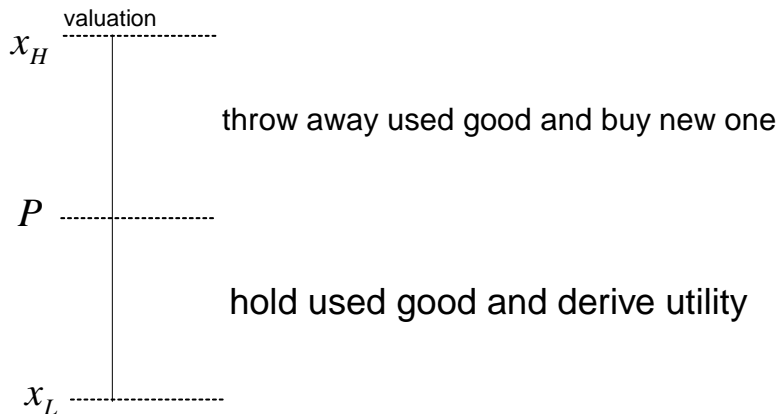
## Environment 2

- $c$ : marginal cost to produce new good
- $d$ : disposal cost of a used good
- $g$ : operation cost for a middleman
- $p$ : price of new good
- $p_S$ : price of used good at which you sell it to a middleman
- $p_B$ : price of used good at which you buy it from a middleman
- $\lambda$ : Poisson rate for a quality shock to come at.
- $x$ : valuation of new good
- $x_L$ : valuation of used good
- $F(x)$ : distribution of  $x \in [x_L, x_H]$ .



- Each consumer buys a new good, derives utility  $x$ .
- Once hit by a quality shock, a consumer chooses to
  - hold used good and derive utility  $x_L$ , or
  - dispose of it, paying cost  $d$ , and buy a new one at price  $p$ .
- Consumers with  $x > P$  throw away used good and buy new one.
- Consumers with  $x \leq P$  hold used good and derive utility until he is hit by the 2nd shock.

# Without Resale (threshold valuation)



# Without Resale (consumers' flow)

## Without Resale (value functions)

$$\begin{aligned}rW_1 &= x + \lambda \max\{W_2 - W_1, -p - d\}, \\rW_2 &= x_L + \lambda(W_1 - W_2 - p - d).\end{aligned}$$

The threshold valuation  $P$  is determined by  $W_1 - W_2 - p - d = 0$  as

$$P = x_L + (r + \lambda)(p + d). \quad (1)$$

The steady state mass of consumers with  $x \leq P$ , holding the used good is

$$n_1 = \frac{F(P)}{2}. \quad (2)$$

# Without Resale (welfare)

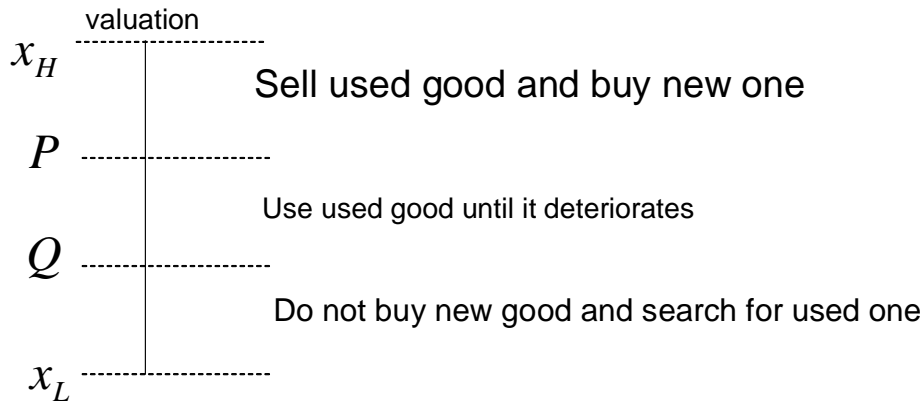
The social planner maximizes the Hamiltonian:

$$\begin{aligned} H(P) &= \int_P^{x_H} [x - \lambda(p + d)]f(x)dx + \int_0^P xn(x)dx \\ &\quad + \int_0^P [x_L - \lambda(p + d)][f(x) - n(x)]dx \\ &\quad + \int_0^P \mu(x)\{\lambda[f(x) - n(x)] - \lambda n(x)\}dx \end{aligned}$$

where  $\mu(x)$  is the costate variable adjoint with state variable  $n(x)$ , and the new good is

**Proposition:** In an economy with no resale, if the new good is supplied at its marginal cost,  $p = c$ , the equilibrium is efficient and the social optimum is decentralized.

# With Resale (threshold valuation)



# With Resale (consumers' flow)

$$\begin{aligned} N_1^S &= 1 - F(P), \quad N_1^{NS} = N_2 = \frac{F(P) - F(Q)}{2}, \\ N_B &= \frac{\lambda F(Q)}{\lambda + \eta}, \quad N_0 = \frac{\eta F(Q)}{\lambda + \eta}. \end{aligned}$$



# Consumers' decision 1

The value function of a consumer with valuation  $x \geq P$ , who buys a new good and sells a used one, satisfies the following value function.

$$rW_1^S(x) = x + \lambda(q_S - p)$$

The value functions of a consumer with valuation  $x \in [Q, P)$ , who buys a new good and uses a used one until it deteriorates, satisfy the following value functions.

$$\begin{aligned} rW_1^{NS}(x) &= x + \lambda(W_2 - W_1) \\ rW_2(x) &= x_L + \lambda[W_1^S(x) - p - W_2 - d], \end{aligned}$$

We obtain

$$W_1^{NS}(x) = \frac{(r + \lambda)x + \lambda x_L - \lambda^2(p + d)}{r(r + 2\lambda)}.$$

The threshold  $P$  is obtained by solving  $W_1^S(x) = W_1^{NS}(x)$ , or

$$P = x_L + (r + \lambda)p - (r + 2\lambda)q_S - \lambda d.$$

## Consumers' decision 2

The value functions of a consumer with  $x \in [x_L, Q)$ , who does not buy the new good, satisfy the following Bellman equations.

$$\begin{aligned}rW_B &= \eta(-q_B + W_0 - W_B) \\rW_0 &= x_L + \lambda(W_B - W_0 - d)\end{aligned}$$

We have

$$W_B = \frac{\eta[x_L - (r + \lambda)q_B - \lambda d]}{r(r + \lambda + \eta)}$$

The threshold valuation  $Q$  satisfies

$$W_1^{NS}(Q) - p = W_B,$$

which leads to

$$Q = -\frac{\eta(r + 2\lambda)}{r + \lambda + \eta}q_B + \frac{\eta - \lambda}{r + \lambda + \eta}x_L + (r + \lambda)p + \frac{\lambda(\lambda - \eta)}{r + \lambda + \eta}d$$

Note that

$$P - Q = \frac{r + 2\lambda}{r + \lambda + \eta} [\eta q_B - (r + \lambda + \eta)q_S + x_L - \lambda d] > 0$$

# Pricing 1 (Resale-sell)

- The instantaneous profit of a middleman is defined by

$$\pi = y(q_B - q_S) - g$$

where  $y$  is the instantaneous quantity sold by a middleman.

$$y = z\eta$$

$z$  is the number of customers who visit a middleman.  $\eta$  is the probability that a customer is served by a middleman.

- The sell-market is frictionless and competitive. Middlemen can buy any amount of the used goods, but do not hold extra inventory, since there are sufficient mass of buyers in the equilibrium.
- The competitive sell-price  $q_S$  is obtained by

$$Mz\eta = \lambda[1 - F(P(q_S))].$$

where  $M$  is the mass of the middlemen.

## Pricing 2 (Resale-buy)

- The middlemen direct buyers' search by posting a buy-price  $q_B$ .
- The probability  $\eta$  that a customer is served by a middleman is derived by Watanabe (2006) as

$$\eta = \frac{\Gamma(k, z)}{\Gamma(k)} + \frac{k}{z} \left[ 1 - \frac{\Gamma(k+1, z)}{\Gamma(k+1)} \right] \quad (3)$$

$$-1 < \frac{\eta'(z)z}{\eta(z)} < 0, \quad \lim_{z \rightarrow \infty} \frac{\eta'(z)z}{\eta(z)} = -1$$

where  $\Gamma(k, z)$  is an incomplete Gamma function,  $k$  is the inventory held by a middleman and  $z$  is the number of customers who visit that middleman.

- example:  $k = 20$ ,  $z = 25$ ,  $50$

$$\frac{\eta'(z)z}{\eta(z)} = \frac{\frac{20}{26} - \frac{20}{25}}{26 - 25} \times \frac{25}{\frac{20}{25}} = -\frac{25}{26}, \quad \frac{\frac{20}{51} - \frac{20}{50}}{51 - 50} \times \frac{50}{\frac{20}{50}} = -\frac{50}{51}$$

- A middleman chooses  $\eta$  and  $q_B$  to maximize his profit

$$\pi = (q_B - q_S)z(\eta)\eta - g.$$

subject to

$$W_B = \frac{\eta[x_L - (r + \lambda)q_B - \lambda d]}{r(r + \lambda + \eta)} > \text{certain level}$$

# The Solution

- The solution is characterized by the tangency of the indifference curve of the buyers to the iso-profit curve of the middlemen, which leads to

$$q_B = \frac{(r + \lambda + \eta)[\zeta(z) + 1]}{\zeta(z)(r + \lambda + \eta) + \eta} q_S + \frac{-x_L + \lambda d}{\zeta(z)(r + \lambda + \eta) + \eta} \quad (4)$$

where  $\zeta(z)$  is the reciprocal of the elasticity of the probability for a customer to be served with respect to the number of customers who visit the middleman.

$$\zeta(z) \equiv \frac{z'(\eta)\eta}{z(\eta)} = \left[ \frac{\eta'(z)z}{\eta(z)} \right]^{-1} < -1, \quad \lim_{z \rightarrow \infty} \zeta(z) = -1$$

- example:  $k = 20$ ,  $z = 25, 50$

$$\frac{z'(\eta)\eta}{z(\eta)} = \frac{26 - 25}{\frac{20}{26} - \frac{20}{25}} \times \frac{\frac{20}{25}}{25} = -\frac{26}{25}, \quad \frac{51 - 50}{\frac{20}{51} - \frac{20}{50}} \times \frac{\frac{20}{50}}{50} = -\frac{51}{50}$$

- There are multiple points at which the indifference curves are tangent to the iso-profit curve.
- However,  $\eta$  is determined by the clearing condition of the resale market as follows.
  - The aggregate number of customer visits equals the mass of used good buyers.

$$Mz = \frac{\lambda F(Q)}{\lambda + \eta} \quad (5)$$

The sell market clears if the aggregate inventory equals the mass of used good sellers.

$$Mz\eta = \lambda[1 - F(P)] \quad (6)$$

(5) and (6) imply

$$\eta = \frac{\lambda[1 - F(P)]}{F(Q) + F(P) - 1} \quad (7)$$

The higher  $P$ , the higher  $q_B$ , and the lower  $\eta$ .  
The higher  $Q$ , the higher  $q_B$ , and the lower  $\eta$ .



# Clearing the Resale Market

The mass of middlemen  $M$  is determined by the zero profit condition.

$$(q_B - q_S) \frac{\lambda[1 - F(P)]}{M} - g = 0 \quad (8)$$

**Definition.** Given the thresholds  $P$ ,  $Q$  and the prices  $p$ ,  $q_S$ , a set  $\{z, \eta, q_B, M\}$  solves an equilibrium of the resale market with middlemen, if it satisfies (7), (3), (4) and (8).

**Proposition.** There is a unique equilibrium of the resale market.



# Full Equilibrium

In order to simplify the analysis, we assume that  $\zeta(z) = -1$ . Then

**Definition:** An equilibrium is a set  $\{q_B, q_S, z, \eta, M, P, Q\}$  that satisfies

$$q_B = \frac{x_L - \lambda d}{r + \lambda}$$

$$z = \frac{g[1 - F(Q) - F(P)](r + \lambda)}{\lambda F(Q)[1 - F(Q)][x_L - (r + \lambda)q_S - \lambda d]}$$

$$\eta = \frac{\lambda[1 - F(P)]}{F(Q) + F(P) - 1}$$

$$(q_B - q_S) \frac{\lambda[1 - F(P)]}{M} - g = 0$$

$$Mz\eta = \lambda[1 - F(P)]$$

$$P = x_L + (r + \lambda)p - (r + 2\lambda)q_S - \lambda d.$$

$$Q = -\frac{\eta(r + 2\lambda)}{r + \lambda + \eta}q_B + \frac{\eta - \lambda}{r + \lambda + \eta}x_L + (r + \lambda)p + \frac{\lambda(\lambda - \eta)}{r + \lambda + \eta}d.$$

# Comparative Statics

## The Baseline

The valuation is uniformly distributed. The baseline simulation is built on the parameters

$$\begin{aligned}x_H &= 20, x_L = 3, F(x) = \frac{x - x_L}{x_H - x_L} \\g &= 10, p = 80, \lambda = 0.1, r = 0.07\end{aligned}$$

# A higher operation cost of the store increases the inventory

$g$	8	9	10	11	12
$q_B$	16.47	16.47	16.47	16.47	16.47
$q_S$	15.83	15.83	15.83	15.83	15.83
$z$	17.19	19.33	21.48	23.63	25.78
$\eta$	.73	.73	.73	.73	.73
$M$	$3.68 \times 10^{-3}$	$3.27 \times 10^{-3}$	$2.95 \times 10^{-3}$	$2.68 \times 10^{-3}$	$2.45 \times 10^{-3}$
$P$	12.12	12.12	12.12	12.12	12.12
$Q$	11.95	11.95	11.95	11.95	11.95

A rise in the disposal cost reduces the inventory and induces more stores to enter.

$d$	2.0	2.1	2.2	2.3
$q_B$	16.47	16.41	16.35	16.29
$q_S$	15.83	14.57	13.40	12.32
$z$	21.48	10.20	8.09	7.28
$\eta$	.73	.53	.42	.35
$M$	$2.95 \times 10^{-3}$	$8.14 \times 10^{-3}$	$12.2 \times 10^{-3}$	$16.2 \times 10^{-3}$
$P$	12.12	12.45	12.75	13.04
$Q$	11.95	11.95	11.96	11.97

# A rise in the price of the new good induces consumers to rush for large stores

$p$	80	85	90	95
$q_B$	16.47	16.47	16.47	16.47
$q_S$	15.83	15.89	15.96	16.02
$z$	21.48	68.35	140.51	256.8
$\eta$	.73	.25	.13	.08
$M$	$2.95 \times 10^{-3}$	$2.375 \times 10^{-3}$	$1.86 \times 10^{-3}$	$1.40 \times 10^{-3}$
$P$	12.12	12.95	13.79	14.65
$Q$	11.95	12.80	13.65	14.50

No resale:

$$P_N = x_L + (r + \lambda)(p + d).$$

With Resale:

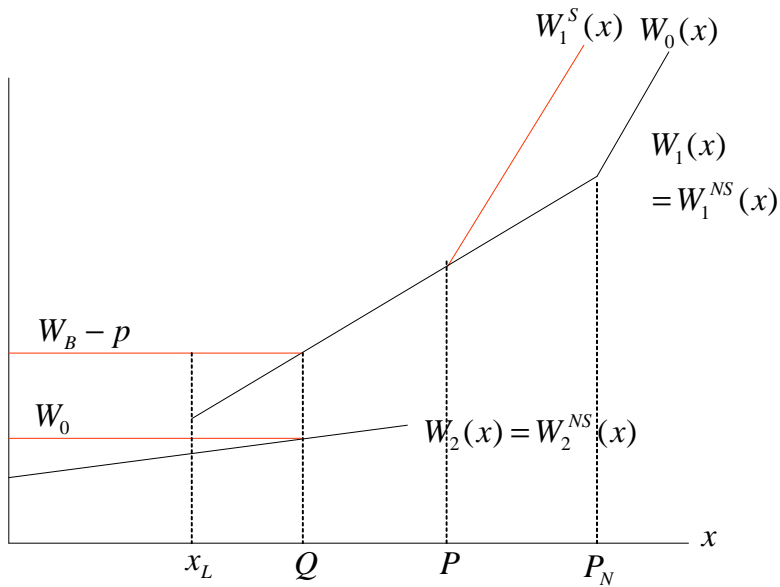
$$P = x_L + (r + \lambda)p - (r + 2\lambda)q_S - \lambda d.$$

$$Q = -\frac{\eta(r + 2\lambda)}{r + \lambda + \eta}q_B + \frac{\eta - \lambda}{r + \lambda + \eta}x_L + (r + \lambda)p + \frac{\lambda(\lambda - \eta)}{r + \lambda + \eta}d$$

## Theorem

*The value of the agents with resale is higher than without resale for  $x < Q$  and  $x > P$ , while it is equal to that without resale for  $x \in [Q, P]$ .*





# Monopoly

By assuming  $p = c$ , we have obtained the welfare results as above. What will occur, if the new good is supplied by a monopolist?

The monopolist profit is written as

$$\pi_M = (p - c)D(p)$$

where

$$D(p) = \lambda[1 - F(P(p))]$$

$$P = \begin{cases} P_N \equiv x_L + (r + \lambda)(p + d) & \text{w/ resale} \\ P_R \equiv x_L + (r + \lambda)p - (r + 2\lambda)q_S - \lambda d. & \text{w/o resale} \end{cases}$$

Obviously, the demand is higher with resale than without resale. What about the margin?

The monopolist chooses  $p$  in order to maximize  $\pi_M$ . The FOC implies an usual mark-up pricing.

$$\frac{p}{p - c} = \varphi(p) \equiv -\frac{pD'(p)}{D(p)} = \frac{pF'(P)(r + \lambda)}{\lambda[1 - F(P(p))]}$$

In the case of the uniform distribution,  $F'(P)$  is constant. Then  $p - c$  increases as  $\lambda[1 - F(P(p))]$  becomes large.

In the case of the exponential distribution with parameter  $\nu$ ,  $F'(P) = \nu e^{-\nu P}$  and  $1 - F(P) = e^{-\nu P}$ . Then

$$p - c = \frac{\lambda}{\nu(r + \lambda)},$$

that is, the margin is constant and the profit is higher with resale.

## Theorem

*Resale yields a higher profit to the monopolist, if the valuation is uniformly or exponentially distributed.*

In the case of the Weibull distribution with shape parameter  $k$  and scale parameter  $\sigma$ ,

$$\frac{F'(P)}{1 - F(P)} = \frac{k}{\sigma} \left( \frac{P}{\sigma} \right)^{k-1}$$





and

$$p - c = \frac{\lambda \sigma^k}{(r + \lambda) k P^{k-1}}.$$





Then  $p - c$  is decreasing in  $P$  if  $k > 1$ . That is, the margin is higher with resale except for the case that the shape parameter  $k$  is less than one.

Thus, for a wider variety of valuation distribution, resale raises the monopolist profit.

# Reference (Industrial Organization)

-  Giacomo Calzolari and Alessandro Pavan (2006), "Monopoly with Resale", *The RAND Journal of Economics*, Vol. 37, No. 2, pp. 362-375.
-  Pascal Courty (2003), Some Economics of Ticket Resale, *Journal of Economic Perspectives*-Volume 17, Number 2-Spring 2003-Pages 85-97.
-  Esteban and Shum (2007) "Durable-goods oligopoly with secondary markets: the case of automobiles," *RAND Journal of Economics* Vol. 38, No. 2, Summer 2007, pp. 1-23.
-  Leslie and Sorensen (2009) "The Welfare Effects of Ticket Resale," mimeo., Stanford University.
-  Moller and Watanabe (2009), "Advance Purchase Discounts and Clearance Sales," forthcoming in *Economic Journal*.

# Reference (Search)

-  Peter Diamond (1982), Aggregate Demand Management in Search Equilibrium, *Journal of Political Economy*, Vol. 90, No. 5, pp. 881-894
-  Imai, Ryoichi (2011), A Search Model of the Resale Market, mimeo.
-  Dale T. Mortensen and Christopher Pissarides (1994), Job Creation and Job Destruction in the theory of Unemployment, *Review of Economic Studies*, volume 61, No. 3, pp.397-415.
-  Watanabe, Makoto (2006), Middlemen, the Visible Market Maker, Working Paper 06-10, Universidad Carlos III de Madrid.