

Production and Investment

Ryoichi Imai
Kyushu University

Production

- The relationship between output and production factors is called “production function.”

Production Function :

$$Y = F(K, L)$$

A production function has a constant returns to scale if,

$$zY = F(zK, zL)$$

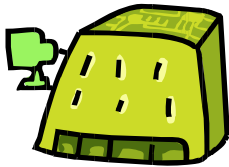
for any positive number z .

Constant Returns to Scale

1 factory

100 workers

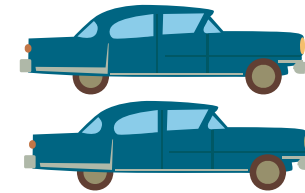
2000 cars / day



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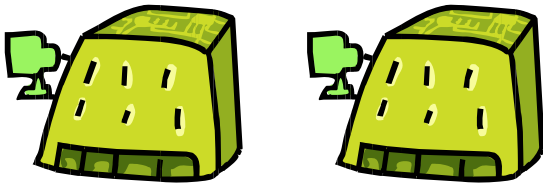
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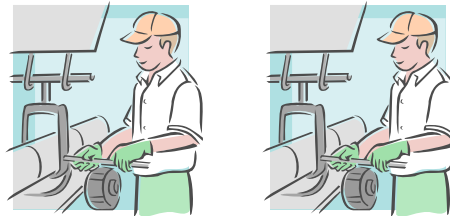
2 factories

200 workers

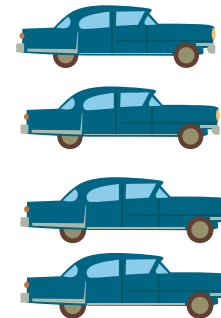
4000 cars / day



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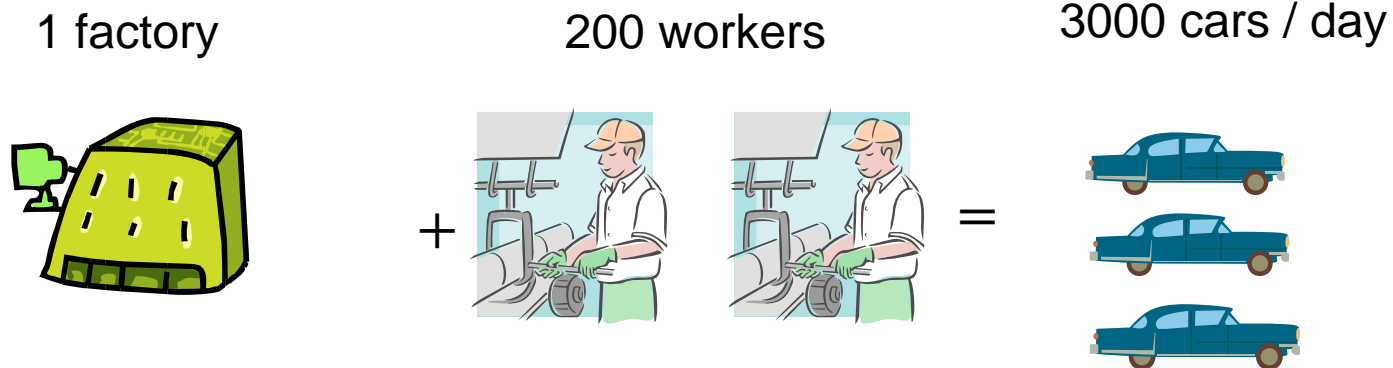
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Production Technology

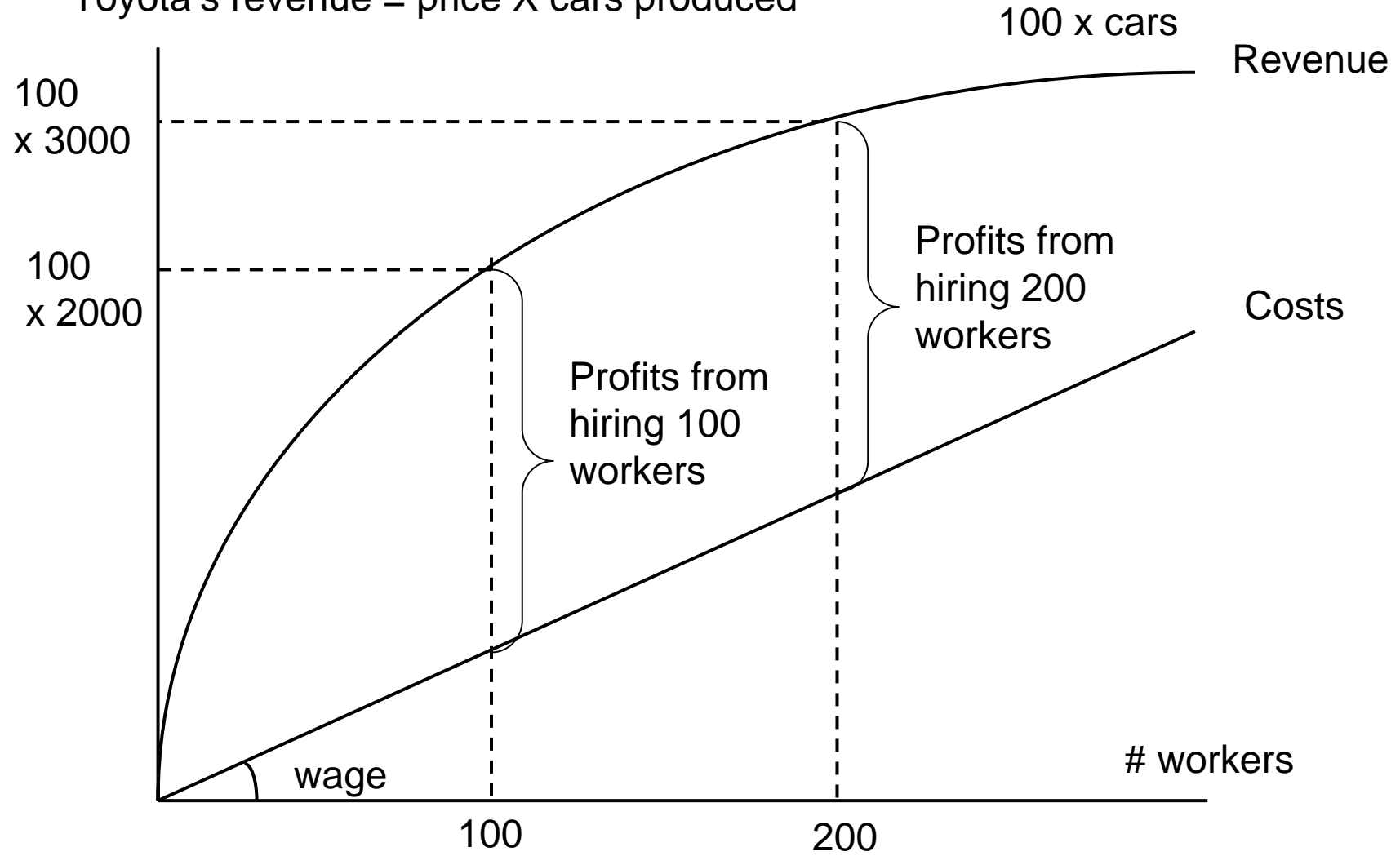
- If all the production factors are flexible to change, the production technology exhibits constant returns to scale (CRS).
- But, if some factors are fixed, the technology shows decreasing returns to the flexible inputs.

Decreasing Returns

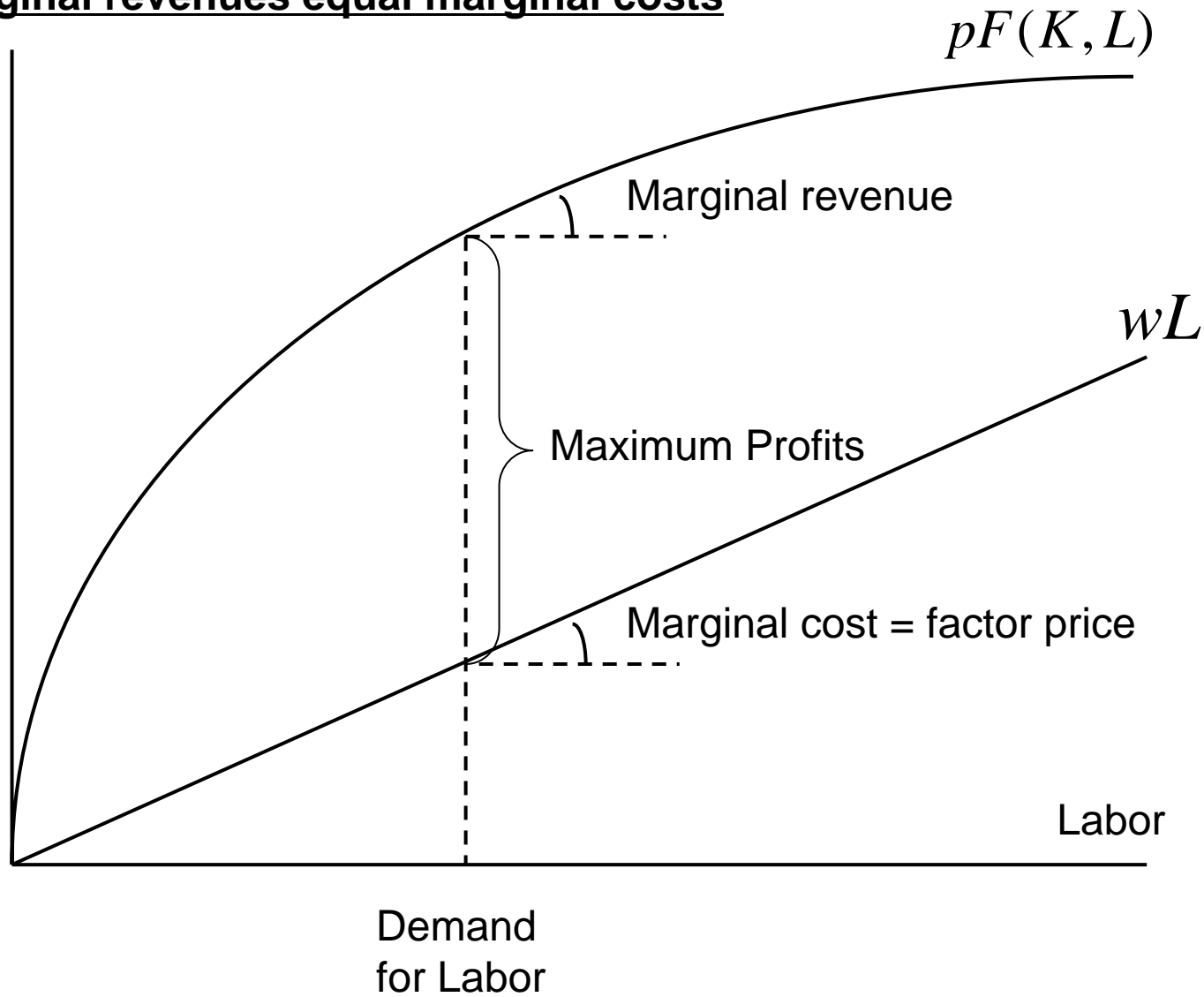


If there is a capacity constraint, production efficiency is limited by space, even though the company hires twice as many workers as before.

Toyota's revenue = price X cars produced



Marginal revenues equal marginal costs



Demand for Factors

- Firms employ each production factor as much as its marginal revenue equals the factor price.
- Marginal revenue = an increase of sales caused by hiring an additional unit of a production factor = slope of the revenue curve.
- Factor price = wage, rent = slope of the cost curve

Investment

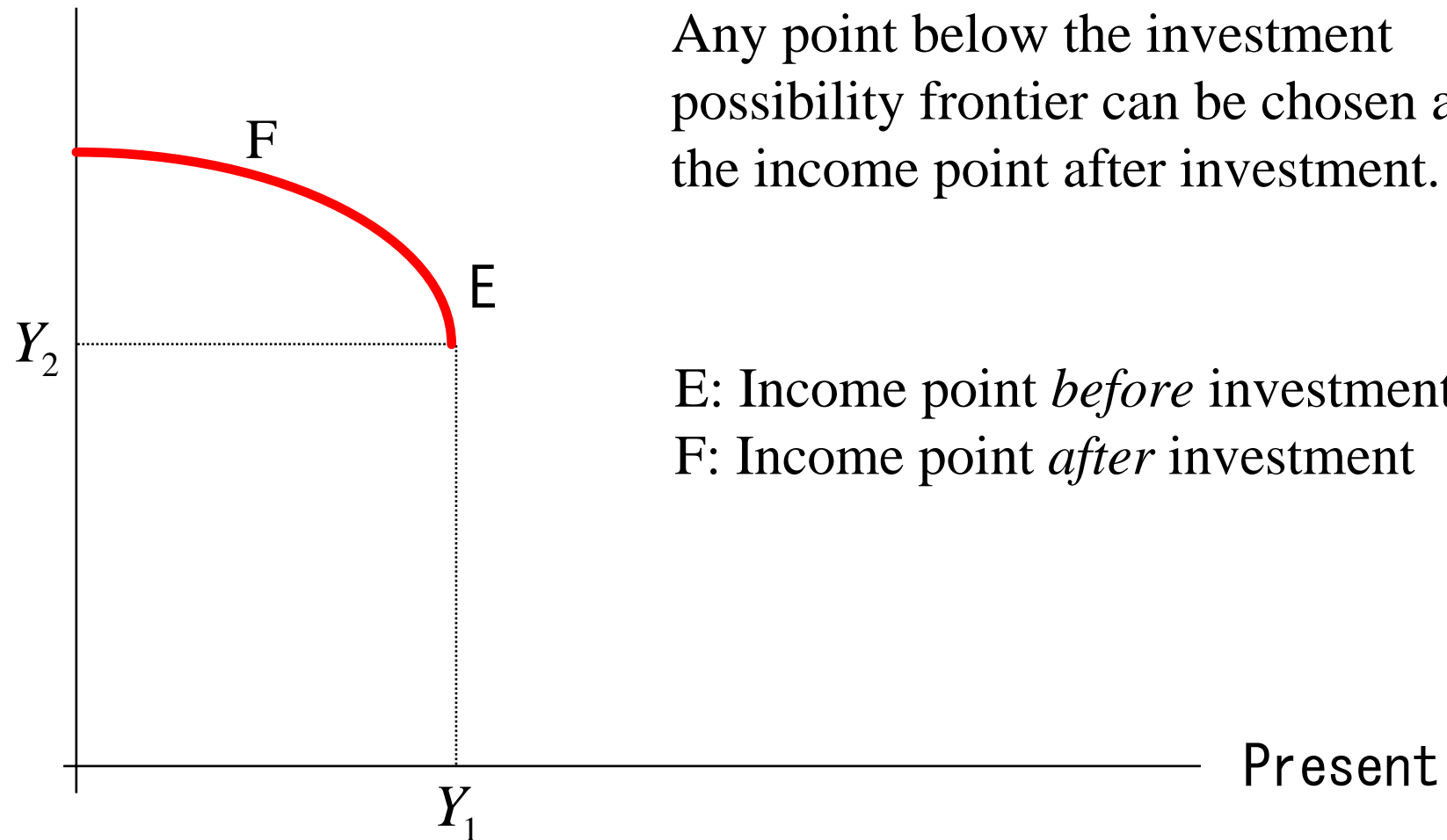
Ryoichi Imai

Investment Creates More Opportunities

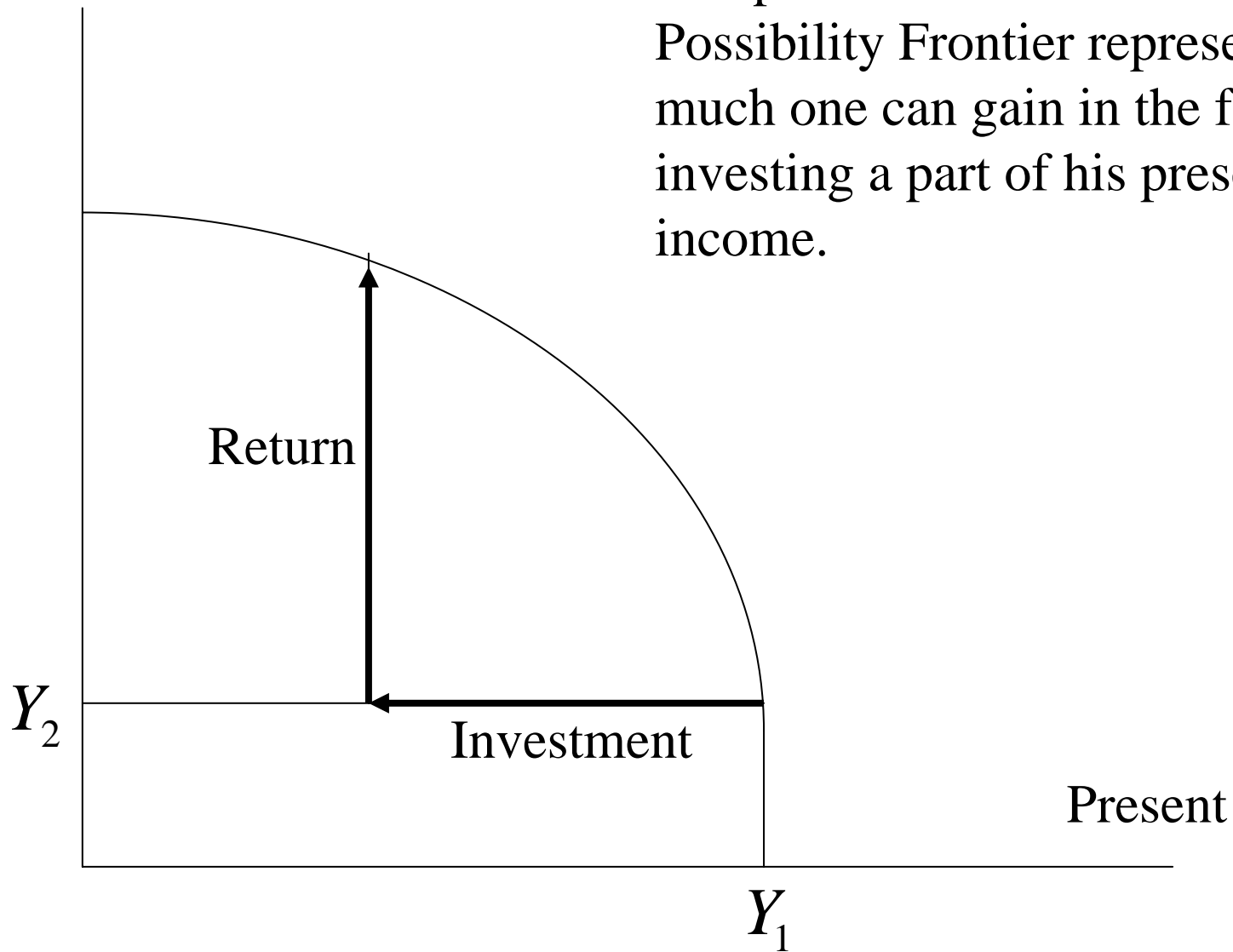
- In the basic model, the income pattern (present - future) is given, and the agent cannot change her/his future income.
- However, the agent can increase her/his future income by investing a portion of her/his present income in a project to increase the future income.

Investment Possibility Frontier

Future



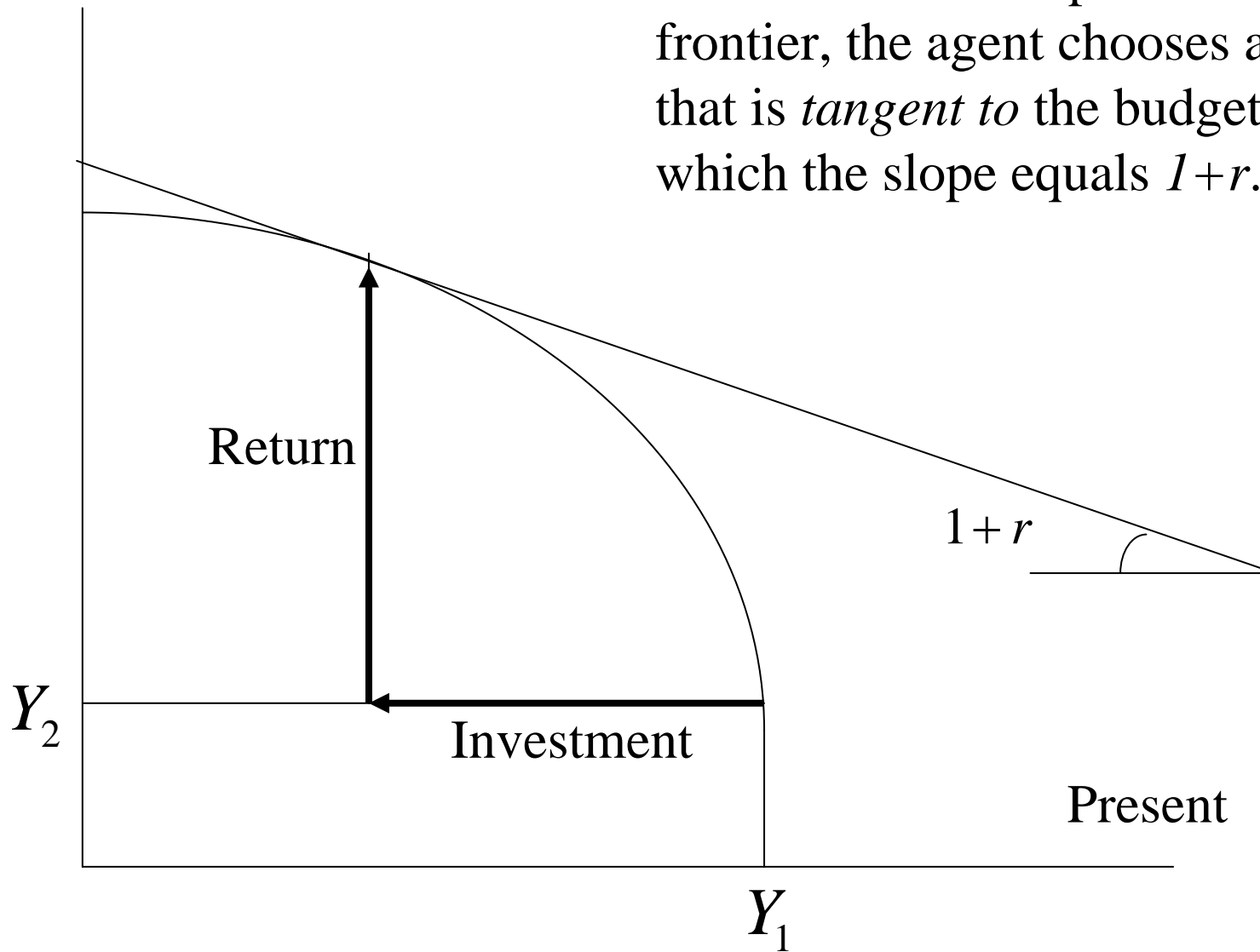
Future



The points on the Investment Possibility Frontier represents how much one can gain in the future by investing a part of his present income.

Future

On the investment possibility frontier, the agent chooses a point that is *tangent* to the budget line, of which the slope equals $1+r$.



The Investment Decision

The agent spends I in the present period, and obtains the outcome $F(I)$ in the future period, which must be discounted by the interest rate r .

The agent problem is to maximize the net gain of investment,

$$\frac{F(I)}{1+r} - I$$

where $F(I)$ denote the output of investment.

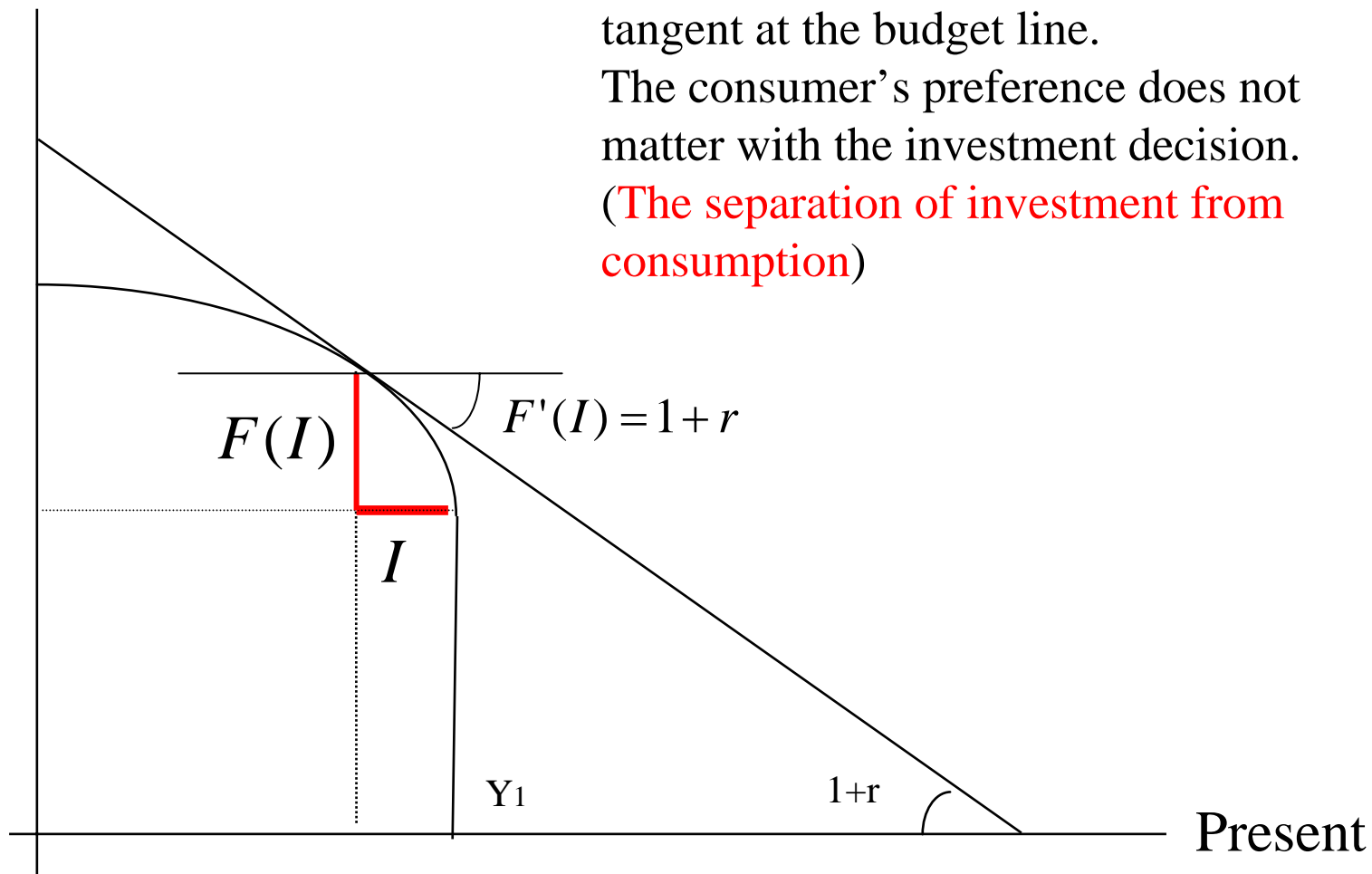
The first - order condition for the maximization is

$$F'(I) = 1 + r$$

In other words, the marginal return of investment equals the gross interest rate (one plus interest rate).

The investment decision is independent of the consumption/saving decision.

Future



A More Mathematical Treatment: The Neoclassical Production Function

Production Function is a relationship between inputs and outputs.

There are n inputs and m outputs.

$$(y_1, y_2, y_3, \dots, y_m) = F(x_1, x_2, x_3, \dots, x_n)$$

The most important example is :

$$y = F(K, L)$$

where K denotes capital and L denotes labor.

The production function shows constant returns to scale if

$$F(\lambda K, \lambda L) = \lambda F(K, L), \quad \text{where } \lambda > 0.$$

The Firm

The purpose of a firm is to maximize its profit, which eventually belongs to households. The firm's profit is

$$\pi = pF(K, L) - rK - wL$$

where w is the wage rate and r is the interest rate.

Assume that π is differentiable in K and L .

Then the first order condition of the profit maximization is

$$\frac{p\partial F(K, L)}{\partial K} = r, \quad \frac{p\partial F(K, L)}{\partial L} = w$$

In other words :

The marginal productivity of capital equals the interest rate.

The marginal productivity of labor equals the wage rate.

The Margin Principle

