

Sample Questions: Macroeconomics II, Spring 2005

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July 11, 2005

In this problem, we extend the original model to include fiscal policy. In the following model, we use the following notations.

b : debts of the government held by the household.

d : debts held by the central bank.

tax : lump-sum tax levied on the household.

g : government consumption

r : real interest rate

tr : money transfer to the household

p : price level

M : money supply

$s = \dot{b}$: purchase of government debts.

i : physical investment

1 The Model

The government bonds evolve according to

$$\dot{b} + \dot{d} + tax = g + r(b + d) + \frac{tr}{p} \quad (1)$$

The sources of financing are listed in the LHS. $\dot{b} + \dot{d}$ denotes the new issues of the government debts, of which b is held by the household and d is held by the central bank. The government uses the revenues for the government consumption g , the interest rate payments $r(b + d)$, and the money transfer to the household tr/p . Money is the measure of value (*numeraire*), so the value of the transfer is tr/p .

The only asset held by the central bank is the government bond. The money supply is provided through the purchase of the government bonds. Therefore,

$$\frac{\dot{M}}{p} = \dot{d} - rd \quad (2)$$

In other words, the (value of) net increase in money supply (\dot{M}/p) is the purchase of new debts net of the receipts of the interests ($\dot{d} - rd$). Combining (1) and (2) we have

$$\dot{b} + \frac{\dot{M}}{p} + tax = g + rb + \frac{tr}{p} \quad (3)$$

The LHS represents the three ways for the government to finance its expenditures: collecting taxes (tax), issuing debts (\dot{b}), and printing money (\dot{M}/p). The RHS represents the three government spending: consumption, interest payments, and money transfers.

The representative household's budget constraint is now written as

$$\frac{\dot{M}}{P} = f(k) - c - tax - \left(i + \frac{i^2}{2}\eta \right) - s + rb + \frac{tr}{P}$$

where $s = \dot{b}$ denotes purchase of new government debts, i is physical investment. $i + i^2\eta/2$ is the *adjustment cost* of physical investment, where $\eta > 0$ is a positive constant. With $M = mP$, we can rewrite this expression as

$$\dot{m} = f(k) - c - tax - \left(i + \frac{i^2}{2}\eta \right) - s + rb + \frac{tr}{P} - \pi m \quad (4)$$

There is no depreciation of government bonds. So the transitional equation of government bonds held by the households is written as

$$\dot{b} = s. \quad (5)$$

The physical capital grows according to

$$\dot{k} = i - \delta k \quad (6)$$

where δ denotes the depreciation rate. The cash-in-advance constraint is written as

$$m \geq c + \varphi \left(i + \frac{i^2}{2}\eta \right) \quad (7)$$

The money transfer follows a rule that $tr = \sigma M$. Then we can rewrite the government budget constraint as

$$\dot{b} + \dot{m} + tax = g + rb + (\sigma - \pi)m \quad (8)$$

The objective function of the household is written as,

$$\int_0^{\infty} u(c)e^{-rt} dt \quad (9)$$

Here we assume that the household discounts the future with the real interest rate r . The representative household maximizes its intertemporal utility (9), subject to the transitional equations of real money balances (4), and the government debts (5), physical capital (6), and the CIA constraint (7). We can write the Hamiltonian as

$$\begin{aligned} H = & u(c) + \lambda \left[f(k) - c - tax - \left(i + \frac{i^2}{2}\eta \right) - s + rb + \frac{tr}{P} - \pi m \right] \\ & + \mu(i - \delta k) + \nu \left[m - c - \varphi \left(i + \frac{i^2}{2}\eta \right) \right] + \xi s \end{aligned}$$

The Maximum Principle consists of the following conditions.

1. FOC:

$$H_c = u'(c) - (\lambda + \nu) = 0 \quad (10)$$

$$H_i = -\lambda - i\eta\lambda + \mu - \nu\varphi(1 + \eta) = 0 \quad (11)$$

$$H_s = -\lambda + \xi = 0 \quad (12)$$

with a complementary slackness condition,

$$0 = \nu \left[m - c - \varphi \left(i + \frac{i^2}{2}\eta \right) \right] \quad (13)$$

which has the following property,

It holds either that $\nu > 0$ and $m - c - \varphi \left(i + \frac{i^2}{2}\eta \right) = 0$, (CIA is binding)

or that $\nu = 0$ and $m - c - \varphi \left(i + \frac{i^2}{2}\eta \right) > 0$. (CIA is not binding)

2. Hamiltonian Dynamics:

$$\dot{\lambda} = r\lambda - H_m = r\lambda + \pi\lambda - \nu \quad (14)$$

$$\dot{\mu} = r\mu - H_k = r\mu - \lambda f'(k) + \delta\mu \quad (15)$$

$$\dot{\xi} = r\xi - H_b = r\xi - \lambda r \quad (16)$$

2 The Steady State Equilibrium

The steady state equilibrium is a set $\{c, i, s, m, k, b, \lambda, \mu, \xi, \nu, tax \text{ or } \sigma\}$ that satisfy the FOCs (10), (11), (12) (13), the Hamiltonian Dynamics (14), (15), (16) and the transitional equations (4), (6), (5), the government budget constraint (8), with $\dot{m} = \dot{k} = \dot{b} = \dot{\lambda} = \dot{\mu} = \dot{\xi} = 0$.

Assume that the CIA constraint is binding, that is, $\nu > 0$. Then, from (11) and (14), we have

$$\nu = \frac{-\lambda - i\eta\lambda + \mu}{\varphi(1 + \eta)} = \lambda(r + \pi)$$

which solves for

$$\mu = [\varphi(1 + \eta)(r + \pi) + (1 + i\eta)]\lambda \quad (17)$$

From (17) and (15), it follows that

$$f'(k) = \frac{\mu}{\lambda}(r + \delta) = (r + \delta)[\varphi(1 + \eta)(r + \pi) + (1 + i\eta)]$$

Using (6), we can rewrite the as expression as

$$f'(k) = \frac{\mu}{\lambda}(r + \delta) = (r + \delta)[\varphi(1 + \eta)(r + \pi) + (1 + \delta\eta k)] \quad (18)$$

The goods market clears if

$$f(k) - c - \left(i + \frac{i^2}{2}\eta\right) - g = f(k) - c - \left(\delta k + \frac{\delta^2 k^2}{2}\eta\right) - g = 0 \quad (19)$$

The money transfer follows $tr = \sigma M$. Then the goods market clearing condition (19) and the steady state ($\dot{m} = 0$ and $\dot{b} = 0$) imply

$$0 = g + rb - tax + (\sigma - \pi)m$$

which is identical to the government's budget constraint (8) in the steady state. The reduced form of the steady state rate of inflation is given by

$$\pi = \sigma - \frac{tax - g - rb}{m} \quad (20)$$

Note that $\pi = \sigma$ if and only if $tax - g - rb = 0$. That is, the steady state rate of inflation equals the nominal money growth rate, if and only if the *primary*

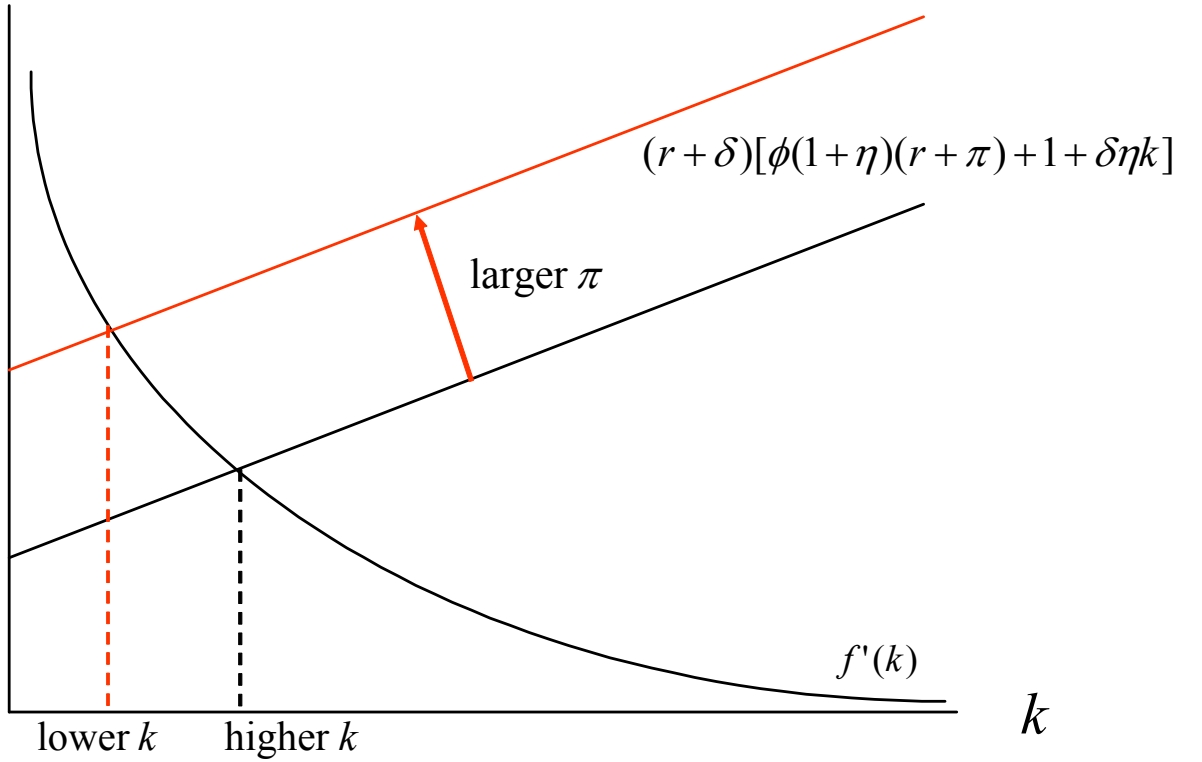


Figure 1:

balance ($tax - g$) just covers the interest payment. If the primary balance is positive, the steady state rate of inflation is less than the nominal money growth rate, and vice versa. The negative primary balance implies that the government deficit is financed by inflation tax. Substituting (20) into (18), we find that the steady state capital satisfies

$$f'(k) = \frac{\mu}{\lambda}(r + \delta) = (r + \delta) \left[\varphi(1 + \eta) \left(r + \sigma - \frac{tax - g - rb}{m} \right) + (1 + \delta\eta k) \right] \quad (21)$$

With this expression, we can explore the fiscal policy's implication for capital accumulation, while the monetary policy is given and a lump-sum tax is levied on households.

3 Questions

1. Does a negative primary balance net of interest payment ($tax - g - rb < 0$) increase or decrease the steady state capital? Consider this problem in Figure 1.
2. Assume now that the primary balance net of interest payment is negative (that is, $tax - g - rb < 0$). Consider an experiment that the central bank replaces Δb of government debts for $\Delta m = \Delta b$ of money in a open market operation. Does this policy increase or decrease the steady state capital?
3. In Q1 and Q2, we just compare the steady states with different policies. What are the potential problems in such a discussion?